Calcul | Test #2 (Unités 2 et 3) | Mme Tarasenco

ktarasenco@wsd1.org

Name: Corrigé Date: ____________

Show all your work!! /33

1. Using the definition of a derivative, determine the derivative of the function $f(x)=\sqrt{x^2+4}$.

/5

$$f'(x)=\lim_{h\to 0}\left[\frac{\sqrt{(x+h)^2+4}-\sqrt{x^2+4}}{h}\cdot\frac{\sqrt{(x+h)^2+4}+\sqrt{x^2+4}}{\sqrt{(x+h)^2+4}+\sqrt{x^2+4}}\right]$$

$$=\lim_{h\to 0}\frac{(x+h)^2+4-(x^2+4)}{h\left(\sqrt{(x+h)^2+4}+\sqrt{x^2+4}\right)}$$

$$=\lim_{h\to 0}\frac{\cancel{x^2}+2xh+h^2+\cancel{4}-\cancel{x^2}-\cancel{4}}{h\left(\sqrt{(x+h)^2+4}+\sqrt{x^2+4}\right)}$$

$$=\lim_{h\to 0}\frac{\cancel{h}(2x+h)}{\cancel{h}\left(\sqrt{(x+h)^2+4}+\sqrt{x^2+4}\right)}$$

$$=\lim_{h\to 0}\frac{2x+h}{\sqrt{(x+h)^2+4}+\sqrt{x^2+4}}$$

$$=\frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+4}}=\boxed{\frac{x}{\sqrt{x^2+4}}}$$

2. **BONUS:** Use the Intermediate-Value Theorem to prove that $\underbrace{x^5-3x^4-2x^3-x+1}_{f(x)}=0$ has at least one solution in the interval [0, 1]. Explain your answer.

/3

$f(0)=0^5-3(0)^4-2(0)^3-0+1=1$

$f(1)=1^5-3(1)^4-2(1)^3-1+1=1-3-2-\cancel{1}+\cancel{1}=-4$

Since $f(0)>0$ and $f(1)<0$, according to the I.V.T. $f(x)$ has at least one solution in the interval $[0,1]$.

3. Explain how the tangent and the derivative are related.

/1 The derivative of the function is the slope of the tangent.

4. Find the equation of the tangent line to the graph of $y = x + \frac{1}{x}$ when $x = 2$.

$y = x + x^{-1}$

/4 $y' = 1 - x^{-2}$

$m(x) = 1 - x^{-2}$

$m(2) = 1 - 2^{-2} = 1 - \frac{1}{4} = \frac{3}{4}$

$x = 2$: $y = 2 + \frac{1}{2} = \frac{5}{2}$

$\therefore \boxed{y - \frac{5}{2} = \frac{3}{4}(x-2)}$

5. Find the derivatives of the following functions.

a) $y = 5x^{-4} + \frac{3}{x} - \frac{5}{x^3} - 8x$ $= 5x^{-4} + 3x^{-1} - 5x^{-3} - 8x$

/2 $\boxed{y' = 5(-4x^{-5}) + 3(-x^{-2}) - 5(-3x^{-4}) - 8}$

b) $y = \sqrt[3]{x^2} - 4x^{-3/4} - \frac{3}{2\sqrt{x}} + 1$ $= x^{2/3} - 4x^{-3/4} - \frac{3}{2}x^{-1/2} + 1$

/2 $\boxed{y' = \frac{2}{3}x^{-1/3} - 4\left(-\frac{3}{4}x^{-7/4}\right) - \frac{3}{2}\left(-\frac{1}{2}x^{-3/2}\right) + 0}$

c) $f(x)=(4x^5-6x^2)(x^3-5x^2+x-2)$

/3 $\boxed{f'(x)=(4(5x^4)-6(2x))(x^3-5x^2+x-2)+(4x^5-6x^2)(3x^2-5(2x)+1-0)}$

d) $f(x)=(2-3\sqrt[4]{x})\left(1-\frac{5}{x}\right) = (2-3x^{1/4})(1-5x^{-1})$

/3 $\boxed{f'(x)=(0-3(\tfrac{1}{4}x^{-3/4}))(1-5x^{-1})+(2-3x^{1/4})(0-5(-x^{-2}))}$

e) $f(x)=\dfrac{2x^4}{8-x^3}$

/3 $\boxed{f'(x)=\dfrac{(2(4x^3))(8-x^3)-(2x^4)(0-3x^2)}{(8-x^3)^2}}$

f) $f(x)=\dfrac{(3x-1)\left(x^6-\frac{2}{\sqrt[5]{x}}\right)}{4x-3} = \dfrac{(3x-1)(x^6-2x^{-1/5})}{4x-3}$

/4

$$\boxed{f'(x)=\frac{[(3-0)(x^6-2x^{-1/5})+(3x-1)(6x^5-2(-\tfrac{1}{5}x^{-6/5})][4x-3)-[(3x-1)(x^6-2x^{-1/5})](4-0)}{(4x-3)^2}}$$

6. If f and g are differentiable functions, prove that $(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$

/4

$$(f(x)g(x))' = \lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x+h)g(x) + f(x+h)g(x) - f(x)g(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)[g(x+h) - g(x)] + g(x)[f(x+h) - f(x)]}{h}$$

$$= \lim_{h\to 0} f(x+h) \cdot \lim_{h\to 0} \frac{g(x+h)-g(x)}{h} + \lim_{h\to 0} g(x) \cdot \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

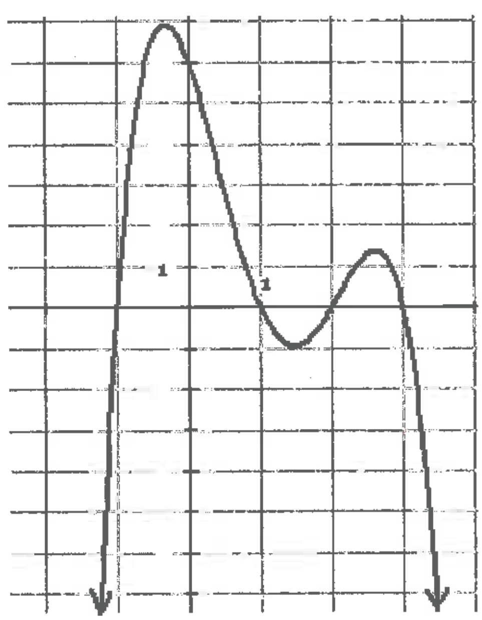
$$= f(x+0) \cdot g'(x) + g(x) \cdot f'(x)$$

$$= f(x)g'(x) + g(x)f'(x)$$

7. Given the graph of f, sketch an approximate graph of the function f'.

/2

f



f' (The ghost graph of f is given)