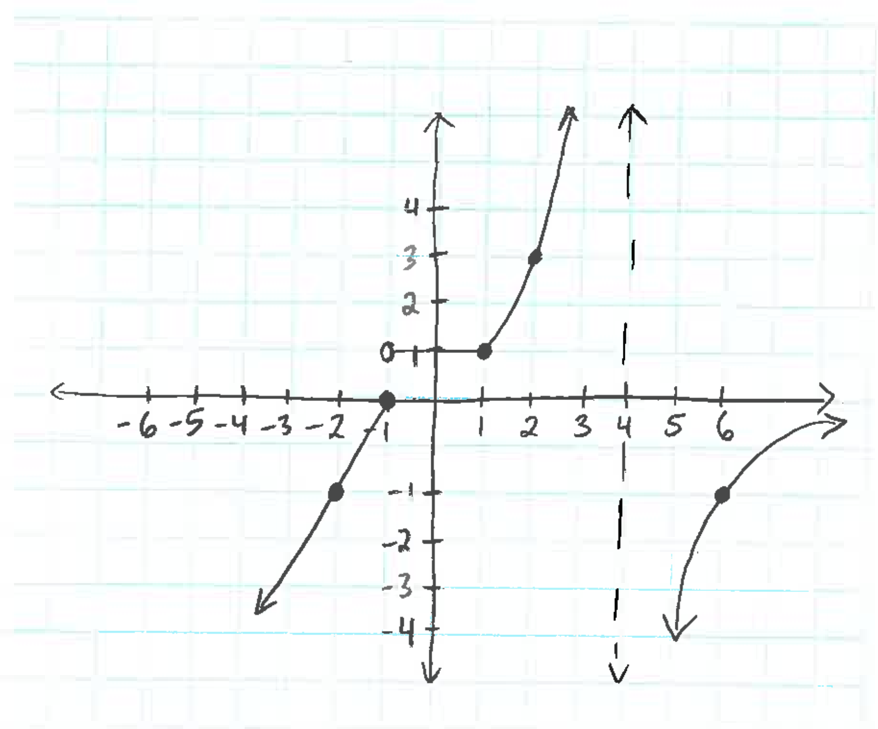
Calcul | Test #1 (Révision et Unité 1) | Mme Tarasenco
ktarasenco@wsd1.org

Name: Corrigé   Date: ____________

Instructions: Show all work
Answer all the questions
Mark values are shown in the left margin

/40

1. The function $h(x)$ is pictured below. Evaluate each function value and limit. State "does not exist (DNE)" if this is the case.



/7

a) $h(-2) = -1$

b) $h(-1) = 0$

c) $h(1) = 1$

d) $h(2) = 3$

e) $h(4) =$ DNE

f) $\lim_{x\to-\infty} h(x) = -\infty$

g) $\lim_{x\to\infty} h(x) = 0$

h) $\lim_{x\to-2} h(x) = -1$

i) $\lim_{x\to-1} h(x) =$ DNE

j) $\lim_{x\to4^-} h(x) = \infty$

k) $\lim_{x\to4^+} h(x) = -\infty$

l) $\lim_{x\to4} h(x) =$ DNE

m) $\lim_{x\to0} h(x) = 1$

n) $\lim_{x\to2} h(x) = 3$

2. Evaluate the following limits:

$$f(x)=\begin{cases} x-1 & if\ x\le -1 \\ x^2+1 & if\ -1<x\le 0 \\ (x+\pi)^2 & if\ x>0 \end{cases}$$

/4

a) $\lim_{x\to -1^-} f(x) = \lim_{x\to -1^-} x-1 = -1-1 = \boxed{-2}$

b) $\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} x^2+1 = (-1)^2+1 = \boxed{2}$

c) $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (x+\pi)^2 = (0+\pi)^2 = \boxed{\pi^2}$

d) $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x^2+1 = 0^2+1 = \boxed{1}$

3. Evaluate the following limits or state "does not exist (DNE)".

a) $\lim_{x\to 3}\dfrac{x-3}{x+3}$

/1

$= \dfrac{3-3}{3+3}$

$= \boxed{0}$

b) $\lim_{x\to e} e^2 = \boxed{e^2}$

/1

c) $\lim_{x\to 2}\dfrac{x^4-16}{x^3-8}$

/3

$\lim_{x\to 2} \dfrac{(x^2+4)(x^2-4)}{(x-2)(x^2+2x+4)}$ 0,5 1

$\lim_{x\to 2} \dfrac{(x^2+4)(x+2)\cancel{(x-2)}}{\cancel{(x-2)}(x^2+2x+4)}$ 0,5

$\lim_{x\to 2} \dfrac{(x^2+4)(x+2)}{x^2+2x+4}$

$= \dfrac{(2^2+4)(2+2)}{2^2+2(2)+4}$ 1

$= \dfrac{8\,(\cancel{4})}{\cancel{12}\,3}$

$= \boxed{\dfrac{8}{3}}$

d) $\lim_{h\to 0}\left[\dfrac{\sqrt{4+h}-2}{h} \cdot \dfrac{\sqrt{4+h}+2}{\sqrt{4+h}+2}\right]$ 1

/3

$\lim_{h\to 0} \dfrac{\cancel{4}+h-\cancel{4}}{h(\sqrt{4+h}+2)}$

$\lim_{h\to 0} \dfrac{\cancel{h}}{\cancel{h}(\sqrt{4+h}+2)}$ 1

$\lim_{h\to 0} \dfrac{1}{\sqrt{4+h}+2}$

$= \dfrac{1}{\sqrt{4+0}+2}$ 1

$= \boxed{\dfrac{1}{4}}$

4. Evaluate the following limits or state "does not exist (DNE)".

a) $\lim_{x\to 1}\left[\frac{x^2-1}{\sqrt{x+3}-2}\cdot\frac{\sqrt{x+3}+2}{\sqrt{x+3}+2}\right]$ ①

/3

$\lim_{x\to 1}\frac{(x+1)(x-1)(\sqrt{x+3}+2)}{x+3-4}$ ①

$\lim_{x\to 1}\frac{(x+1)\cancel{(x-1)}(\sqrt{x+3}+2)}{\cancel{x-1}}$

$\lim_{x\to 1}(x+1)(\sqrt{x+3}+2)$

$=(1+1)(\sqrt{1+3}+2)$ ①

$=2(4)$

$=\boxed{8}$

b) $\lim_{x\to 1^-}\frac{x+4}{|x-1|}$

$0{,}99-1=\ominus$

/2

$\lim_{x\to 1^-}\frac{x+4}{-(x-1)}$

$\lim_{x\to 1^-}\frac{x+4}{-x+1}$

$\frac{0{,}99+4}{-0{,}99+1}$

$=\frac{\oplus}{\text{petit }\oplus}$ ①

$=\boxed{\infty}$ ①

c) $\lim_{x\to 2^+}\frac{x}{(2-x)^3}$

$2-2{,}1=\text{petit }\ominus$

/2

$=\frac{\oplus}{\text{petit }\ominus}$ ①

$=\boxed{-\infty}$ ①

5. Find m so that $f(x)=\begin{cases} x-m & if \quad x<3 \\ 1-mx & if \quad x\geq 3 \end{cases}$ is a continuous function. Use the definition of continuity to show your work.

For the function to be continuous, $\lim_{x\to 3^-} f(x)$ must equal $\lim_{x\to 3^+} f(x)$.

$\lim_{x\to 3^-} x-m = 3-m$ $\qquad$ $\lim_{x\to 3^+} 1-mx = 1-3m$

/3

$\therefore 3-m = 1-3m$ ①

$2m = -2$ ①

$m = -1$

$\lim_{x\to 3^-} x-(-1) = 3+1 = 4$ $\qquad$ $\lim_{x\to 3^+} 1-(-1)x = 1+3 = 4$ $\qquad$ $\therefore \lim_{x\to 3} f(x) = 4$

$f(3)$ must equal $\lim_{x\to 3} f(x)$. $\quad f(3) = 1-(-1)(3) = 1+3 = 4$. ①

$\therefore f(x)$ is continuous for $\boxed{m=-1}$ at $x=3$

6. Evaluate the limits at infinity.

a) $\lim\limits_{x\to -\infty} \dfrac{x}{\sqrt{5x^2-2x}-x}$

$$\lim_{x\to\infty} \frac{-x}{\sqrt{5x^2+2x}+x} \quad ①$$

/3

$$\lim_{x\to\infty} \frac{-x}{\sqrt{x^2}\sqrt{5+2/x}+x}$$

①

$$\lim_{x\to\infty} \frac{-x}{x\sqrt{5+2/x}+x}$$

$$\lim_{x\to\infty} \frac{-\cancel{x}}{\cancel{x}(\sqrt{5+2/x}+1)}$$

$$\lim_{x\to\infty} \frac{-1}{\sqrt{5+2/x}+1} \quad ①$$

$$= \frac{-1}{\sqrt{5+0}+1} = \boxed{\frac{-1}{\sqrt{5}+1}}$$

b) $\lim\limits_{x\to\infty} \dfrac{3x+2\sqrt{x}}{1-x}$

/2

$$\lim_{x\to\infty} \frac{\frac{3x}{x}+\frac{2\sqrt{x}}{x}}{\frac{1}{x}-\frac{x}{x}} \quad ①$$

$$\lim_{x\to\infty} \frac{3+\frac{2}{\sqrt{x}}}{\frac{1}{x}-1}$$

$$= \frac{3+0}{0-1} \quad ①$$

$$= \boxed{-3}$$

c) $\lim_{x\to-\infty} \frac{x^7-1}{3x^3-5}$

$\lim_{x\to\infty} \frac{-x^7-1}{-3x^3-5}$ 1

/2 $\lim_{x\to\infty} \frac{-\frac{x^7}{x^3}-\frac{1}{x^3}}{-\frac{3x^3}{x^3}-\frac{5}{x^3}}$ 1

$\lim_{x\to\infty} \frac{-x^4-\frac{1}{x^3}}{-3-\frac{5}{x^3}}$

$= \frac{-\infty-0}{-3-0}$

$= \boxed{\infty}$

d) $\lim_{x\to\infty} \frac{3x^3-2x^2+1}{4-2x+6x^3}$

/1

$= \boxed{\frac{1}{2}}$

7. Use the Limit Squeeze Theorem to find $\lim_{x\to 0} f(x)$ if $2-x^2 \le f(x) \le 2\cos x$ for all x.

$\lim_{x\to 0} 2-x^2 = 2-0^2 = 2$ 1

/3 $\lim_{x\to 0} 2\cos x = 2\cos(0) = 2(1) = 2$ 1

Therefore, according to the Limit Squeeze Theorem, $\lim_{x\to 0} f(x) = 2$.

1

#8. Find the domain of

$$f(x) = \sqrt{\frac{(x-2)(x+3)}{(x-1)}}$$

$\hookrightarrow x \neq 1$

		-3		1		2		1
$x-2$	$-$		$-$		$-$		$+$	
$x+3$	$-$		$+$		$+$		$+$	
$x-1$	$-$		$-$		$+$		$+$	
$\frac{(x-2)(x+3)}{x-1}$	$-$		$+$		$-$		$+$	1

$[-3, 1) \cup [2, \infty)$ 1