3.4 The Chain Rule

1. Let $u = g(x) = 4x$ and $y = f(u) = \sin u$. Then $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx} = (\cos u)(4) = 4\cos 4x$.

3. Let $u = g(x) = 1 - x^2$ and $y = f(u) = u^{10}$. Then $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx} = (10u^9)(-2x) = -20x(1-x^2)^9$.

5. Let $u = g(x) = \sqrt{x}$ and $y = f(u) = e^u$. Then $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx} = (e^u)\left(\frac{1}{2}x^{-1/2}\right) = e^{\sqrt{x}} \cdot \dfrac{1}{2\sqrt{x}} = \dfrac{e^{\sqrt{x}}}{2\sqrt{x}}$.

7. $F(x) = (x^4 + 3x^2 - 2)^5 \Rightarrow F'(x) = 5(x^4 + 3x^2 - 2)^4 \cdot \dfrac{d}{dx}(x^4 + 3x^2 - 2) = 5(x^4 + 3x^2 - 2)^4(4x^3 + 6x)$
$\left[\text{or } 10x(x^4 + 3x^2 - 2)^4(2x^2 + 3)\right]$

9. $F(x) = \sqrt[4]{1 + 2x + x^3} = (1 + 2x + x^3)^{1/4} \Rightarrow$

$$F'(x) = \tfrac{1}{4}(1 + 2x + x^3)^{-3/4} \cdot \frac{d}{dx}(1 + 2x + x^3) = \frac{1}{4(1 + 2x + x^3)^{3/4}} \cdot (2 + 3x^2) = \frac{2 + 3x^2}{4(1 + 2x + x^3)^{3/4}}$$
$$= \frac{2 + 3x^2}{4\sqrt[4]{(1 + 2x + x^3)^3}}$$

11. $g(t) = \dfrac{1}{(t^4 + 1)^3} = (t^4 + 1)^{-3} \Rightarrow g'(t) = -3(t^4 + 1)^{-4}(4t^3) = -12t^3(t^4 + 1)^{-4} = \dfrac{-12t^3}{(t^4 + 1)^4}$

13. $y = \cos(a^3 + x^3) \Rightarrow y' = -\sin(a^3 + x^3) \cdot 3x^2$ [a^3 is just a constant] $= -3x^2 \sin(a^3 + x^3)$

15. $y = xe^{-kx} \Rightarrow y' = x\left[e^{-kx}(-k)\right] + e^{-kx} \cdot 1 = e^{-kx}(-kx + 1)$ $\left[\text{or } (1 - kx)e^{-kx}\right]$

17. $f(x) = (2x - 3)^4(x^2 + x + 1)^5 \Rightarrow$

$$\begin{aligned} f'(x) &= (2x - 3)^4 \cdot 5(x^2 + x + 1)^4(2x + 1) + (x^2 + x + 1)^5 \cdot 4(2x - 3)^3 \cdot 2 \\ &= (2x - 3)^3(x^2 + x + 1)^4[(2x - 3) \cdot 5(2x + 1) + (x^2 + x + 1) \cdot 8] \\ &= (2x - 3)^3(x^2 + x + 1)^4(20x^2 - 20x - 15 + 8x^2 + 8x + 8) = (2x - 3)^3(x^2 + x + 1)^4(28x^2 - 12x - 7) \end{aligned}$$

19. $h(t) = (t + 1)^{2/3}(2t^2 - 1)^3 \Rightarrow$

$$\begin{aligned} h'(t) &= (t + 1)^{2/3} \cdot 3(2t^2 - 1)^2 \cdot 4t + (2t^2 - 1)^3 \cdot \tfrac{2}{3}(t + 1)^{-1/3} = \tfrac{2}{3}(t + 1)^{-1/3}(2t^2 - 1)^2[18t(t + 1) + (2t^2 - 1)] \\ &= \tfrac{2}{3}(t + 1)^{-1/3}(2t^2 - 1)^2(20t^2 + 18t - 1) \end{aligned}$$

21. $y = \left(\dfrac{x^2 + 1}{x^2 - 1}\right)^3 \Rightarrow$

$$\begin{aligned} y' &= 3\left(\frac{x^2 + 1}{x^2 - 1}\right)^2 \cdot \frac{d}{dx}\left(\frac{x^2 + 1}{x^2 - 1}\right) = 3\left(\frac{x^2 + 1}{x^2 - 1}\right)^2 \cdot \frac{(x^2 - 1)(2x) - (x^2 + 1)(2x)}{(x^2 - 1)^2} \\ &= 3\left(\frac{x^2 + 1}{x^2 - 1}\right)^2 \cdot \frac{2x[x^2 - 1 - (x^2 + 1)]}{(x^2 - 1)^2} = 3\left(\frac{x^2 + 1}{x^2 - 1}\right)^2 \cdot \frac{2x(-2)}{(x^2 - 1)^2} = \frac{-12x(x^2 + 1)^2}{(x^2 - 1)^4} \end{aligned}$$

23. $y = \sqrt{1 + 2e^{3x}} \Rightarrow y' = \dfrac{1}{2}(1 + 2e^{3x})^{-1/2}\dfrac{d}{dx}(1 + 2e^{3x}) = \dfrac{1}{2\sqrt{1 + 2e^{3x}}}(2e^{3x} \cdot 3) = \dfrac{3e^{3x}}{\sqrt{1 + 2e^{3x}}}$

25. Using Formula 5 and the Chain Rule, $y = 5^{-1/x} \Rightarrow y' = 5^{-1/x}(\ln 5)\left[-1 \cdot (-x^{-2})\right] = 5^{-1/x}(\ln 5)/x^2$

27. $y = \dfrac{r}{\sqrt{r^2+1}} \quad \Rightarrow$

$$y' = \frac{\sqrt{r^2+1}\,(1) - r\cdot\frac{1}{2}(r^2+1)^{-1/2}(2r)}{\left(\sqrt{r^2+1}\right)^2} = \frac{\sqrt{r^2+1} - \dfrac{r^2}{\sqrt{r^2+1}}}{\left(\sqrt{r^2+1}\right)^2} = \frac{\dfrac{\sqrt{r^2+1}\,\sqrt{r^2+1} - r^2}{\sqrt{r^2+1}}}{\left(\sqrt{r^2+1}\right)^2}$$

$$= \frac{(r^2+1) - r^2}{\left(\sqrt{r^2+1}\right)^3} = \frac{1}{(r^2+1)^{3/2}} \text{ or } (r^2+1)^{-3/2}.$$

Another solution: Write y as a product and make use of the Product Rule. $y = r(r^2+1)^{-1/2} \quad \Rightarrow$

$y' = r\cdot -\frac{1}{2}(r^2+1)^{-3/2}(2r) + (r^2+1)^{-1/2}\cdot 1 = (r^2+1)^{-3/2}[-r^2 + (r^2+1)^1] = (r^2+1)^{-3/2}(1) = (r^2+1)^{-3/2}$.

The step that students usually have trouble with is factoring out $(r^2+1)^{-3/2}$. But this is no different than factoring out x^2 from $x^2 + x^5$; that is, we are just factoring out a factor with the *smallest* exponent that appears on it. In this case, $-\frac{3}{2}$ is smaller than $-\frac{1}{2}$.

29. By (9), $F(t) = e^{t\sin 2t} \quad \Rightarrow \quad F'(t) = e^{t\sin 2t}(t\sin 2t)' = e^{t\sin 2t}(t\cdot 2\cos 2t + \sin 2t\cdot 1) = e^{t\sin 2t}(2t\cos 2t + \sin 2t)$

31. $y = \sin(\tan 2x) \quad \Rightarrow \quad y' = \cos(\tan 2x)\cdot \dfrac{d}{dx}(\tan 2x) = \cos(\tan 2x)\cdot \sec^2(2x)\cdot \dfrac{d}{dx}(2x) = 2\cos(\tan 2x)\sec^2(2x)$

33. Using Formula 5 and the Chain Rule, $y = 2^{\sin \pi x} \quad \Rightarrow$

$$y' = 2^{\sin\pi x}(\ln 2)\cdot \frac{d}{dx}(\sin \pi x) = 2^{\sin\pi x}(\ln 2)\cdot\cos\pi x\cdot\pi = 2^{\sin\pi x}(\pi\ln 2)\cos\pi x$$

35. $y = \cos\left(\dfrac{1-e^{2x}}{1+e^{2x}}\right) \quad \Rightarrow$

$$y' = -\sin\left(\frac{1-e^{2x}}{1+e^{2x}}\right)\cdot\frac{d}{dx}\left(\frac{1-e^{2x}}{1+e^{2x}}\right) = -\sin\left(\frac{1-e^{2x}}{1+e^{2x}}\right)\cdot\frac{(1+e^{2x})(-2e^{2x}) - (1-e^{2x})(2e^{2x})}{(1+e^{2x})^2}$$

$$= -\sin\left(\frac{1-e^{2x}}{1+e^{2x}}\right)\cdot\frac{-2e^{2x}\left[(1+e^{2x}) + (1-e^{2x})\right]}{(1+e^{2x})^2} = -\sin\left(\frac{1-e^{2x}}{1+e^{2x}}\right)\cdot\frac{-2e^{2x}(2)}{(1+e^{2x})^2} = \frac{4e^{2x}}{(1+e^{2x})^2}\cdot\sin\left(\frac{1-e^{2x}}{1+e^{2x}}\right)$$

37. $y = \cot^2(\sin\theta) = [\cot(\sin\theta)]^2 \quad \Rightarrow$

$$y' = 2[\cot(\sin\theta)]\cdot\frac{d}{d\theta}[\cot(\sin\theta)] = 2\cot(\sin\theta)\cdot[-\csc^2(\sin\theta)\cdot\cos\theta] = -2\cos\theta\,\cot(\sin\theta)\,\csc^2(\sin\theta)$$

39. $f(t) = \tan(e^t) + e^{\tan t} \quad \Rightarrow \quad f'(t) = \sec^2(e^t)\cdot\dfrac{d}{dt}(e^t) + e^{\tan t}\cdot\dfrac{d}{dt}(\tan t) = \sec^2(e^t)\cdot e^t + e^{\tan t}\cdot\sec^2 t$

41. $f(t) = \sin^2\left(e^{\sin^2 t}\right) = \left[\sin\left(e^{\sin^2 t}\right)\right]^2 \quad \Rightarrow$

$$f'(t) = 2\left[\sin\left(e^{\sin^2 t}\right)\right]\cdot\frac{d}{dt}\sin\left(e^{\sin^2 t}\right) = 2\sin\left(e^{\sin^2 t}\right)\cdot\cos\left(e^{\sin^2 t}\right)\cdot\frac{d}{dt}e^{\sin^2 t}$$

$$= 2\sin\left(e^{\sin^2 t}\right)\cos\left(e^{\sin^2 t}\right)\cdot e^{\sin^2 t}\cdot\frac{d}{dt}\sin^2 t = 2\sin\left(e^{\sin^2 t}\right)\cos\left(e^{\sin^2 t}\right)e^{\sin^2 t}\cdot 2\sin t\,\cos t$$

$$= 4\sin\left(e^{\sin^2 t}\right)\cos\left(e^{\sin^2 t}\right)e^{\sin^2 t}\sin t\,\cos t$$

43. $g(x) = (2ra^{rx} + n)^p \quad\Rightarrow$

$$g'(x) = p(2ra^{rx} + n)^{p-1} \cdot \frac{d}{dx}(2ra^{rx} + n) = p(2ra^{rx} + n)^{p-1} \cdot 2ra^{rx}(\ln a) \cdot r = 2r^2 p(\ln a)(2ra^{rx} + n)^{p-1} a^{rx}$$

45. $y = \cos\sqrt{\sin(\tan \pi x)} = \cos(\sin(\tan \pi x))^{1/2} \quad\Rightarrow$

$$y' = -\sin(\sin(\tan \pi x))^{1/2} \cdot \frac{d}{dx}(\sin(\tan \pi x))^{1/2} = -\sin(\sin(\tan \pi x))^{1/2} \cdot \tfrac{1}{2}(\sin(\tan \pi x))^{-1/2} \cdot \frac{d}{dx}(\sin(\tan \pi x))$$

$$= \frac{-\sin\sqrt{\sin(\tan \pi x)}}{2\sqrt{\sin(\tan \pi x)}} \cdot \cos(\tan \pi x) \cdot \frac{d}{dx}\tan \pi x = \frac{-\sin\sqrt{\sin(\tan \pi x)}}{2\sqrt{\sin(\tan \pi x)}} \cdot \cos(\tan \pi x) \cdot \sec^2(\pi x) \cdot \pi$$

$$= \frac{-\pi \cos(\tan \pi x)\sec^2(\pi x)\sin\sqrt{\sin(\tan \pi x)}}{2\sqrt{\sin(\tan \pi x)}}$$

47. $y = \cos(x^2) \quad\Rightarrow\quad y' = -\sin(x^2) \cdot 2x = -2x\sin(x^2) \quad\Rightarrow$

$$y'' = -2x\cos(x^2) \cdot 2x + \sin(x^2) \cdot (-2) = -4x^2\cos(x^2) - 2\sin(x^2)$$

49. $y = e^{\alpha x}\sin \beta x \quad\Rightarrow\quad y' = e^{\alpha x} \cdot \beta\cos\beta x + \sin\beta x \cdot \alpha e^{\alpha x} = e^{\alpha x}(\beta\cos\beta x + \alpha\sin\beta x) \quad\Rightarrow$

$$y'' = e^{\alpha x}(-\beta^2\sin\beta x + \alpha\beta\cos\beta x) + (\beta\cos\beta x + \alpha\sin\beta x) \cdot \alpha e^{\alpha x}$$

$$= e^{\alpha x}(-\beta^2\sin\beta x + \alpha\beta\cos\beta x + \alpha\beta\cos\beta x + \alpha^2\sin\beta x) = e^{\alpha x}(\alpha^2\sin\beta x - \beta^2\sin\beta x + 2\alpha\beta\cos\beta x)$$

$$= e^{\alpha x}\left[(\alpha^2 - \beta^2)\sin\beta x + 2\alpha\beta\cos\beta x\right]$$

51. $y = (1 + 2x)^{10} \quad\Rightarrow\quad y' = 10(1 + 2x)^9 \cdot 2 = 20(1 + 2x)^9$.

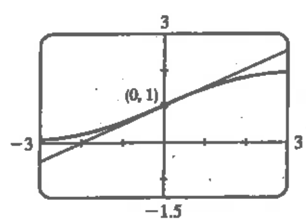
At $(0, 1)$, $y' = 20(1 + 0)^9 = 20$, and an equation of the tangent line is $y - 1 = 20(x - 0)$, or $y = 20x + 1$.

53. $y = \sin(\sin x) \quad\Rightarrow\quad y' = \cos(\sin x) \cdot \cos x$. At $(\pi, 0)$, $y' = \cos(\sin \pi) \cdot \cos \pi = \cos(0) \cdot (-1) = 1(-1) = -1$, and an equation of the tangent line is $y - 0 = -1(x - \pi)$, or $y = -x + \pi$.

55. (a) $y = \dfrac{2}{1 + e^{-x}} \quad\Rightarrow\quad y' = \dfrac{(1 + e^{-x})(0) - 2(-e^{-x})}{(1 + e^{-x})^2} = \dfrac{2e^{-x}}{(1 + e^{-x})^2}$.

At $(0, 1)$, $y' = \dfrac{2e^0}{(1 + e^0)^2} = \dfrac{2(1)}{(1 + 1)^2} = \dfrac{2}{2^2} = \dfrac{1}{2}$. So an equation of the tangent line is $y - 1 = \frac{1}{2}(x - 0)$ or $y = \frac{1}{2}x + 1$.

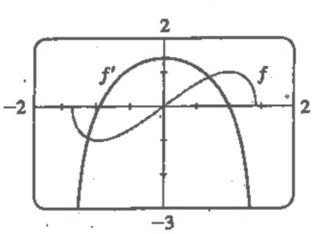
(b)



57. (a) $f(x) = x\sqrt{2 - x^2} = x(2 - x^2)^{1/2} \quad\Rightarrow$

$$f'(x) = x \cdot \tfrac{1}{2}(2 - x^2)^{-1/2}(-2x) + (2 - x^2)^{1/2} \cdot 1 = (2 - x^2)^{-1/2}\left[-x^2 + (2 - x^2)\right] = \frac{2 - 2x^2}{\sqrt{2 - x^2}}$$

(b)



$f' = 0$ when f has a horizontal tangent line, f' is negative when f is decreasing, and f' is positive when f is increasing.

59. For the tangent line to be horizontal, $f'(x) = 0$. $f(x) = 2\sin x + \sin^2 x \Rightarrow f'(x) = 2\cos x + 2\sin x\cos x = 0 \Leftrightarrow$ $2\cos x(1+\sin x) = 0 \Leftrightarrow \cos x = 0$ or $\sin x = -1$, so $x = \frac{\pi}{2} + 2n\pi$ or $\frac{3\pi}{2} + 2n\pi$, where n is any integer. Now $f\left(\frac{\pi}{2}\right) = 3$ and $f\left(\frac{3\pi}{2}\right) = -1$, so the points on the curve with a horizontal tangent are $\left(\frac{\pi}{2} + 2n\pi, 3\right)$ and $\left(\frac{3\pi}{2} + 2n\pi, -1\right)$, where n is any integer.

61. $F(x) = f(g(x)) \Rightarrow F'(x) = f'(g(x)) \cdot g'(x)$, so $F'(5) = f'(g(5)) \cdot g'(5) = f'(-2) \cdot 6 = 4 \cdot 6 = 24$

63. (a) $h(x) = f(g(x)) \Rightarrow h'(x) = f'(g(x)) \cdot g'(x)$, so $h'(1) = f'(g(1)) \cdot g'(1) = f'(2) \cdot 6 = 5 \cdot 6 = 30$.

(b) $H(x) = g(f(x)) \Rightarrow H'(x) = g'(f(x)) \cdot f'(x)$, so $H'(1) = g'(f(1)) \cdot f'(1) = g'(3) \cdot 4 = 9 \cdot 4 = 36$.

65. (a) $u(x) = f(g(x)) \Rightarrow u'(x) = f'(g(x))g'(x)$. So $u'(1) = f'(g(1))g'(1) = f'(3)g'(1)$. To find $f'(3)$, note that f is linear from $(2,4)$ to $(6,3)$, so its slope is $\dfrac{3-4}{6-2} = -\dfrac{1}{4}$. To find $g'(1)$, note that g is linear from $(0,6)$ to $(2,0)$, so its slope is $\dfrac{0-6}{2-0} = -3$. Thus, $f'(3)g'(1) = \left(-\frac{1}{4}\right)(-3) = \frac{3}{4}$.

(b) $v(x) = g(f(x)) \Rightarrow v'(x) = g'(f(x))f'(x)$. So $v'(1) = g'(f(1))f'(1) = g'(2)f'(1)$, which does not exist since $g'(2)$ does not exist.

(c) $w(x) = g(g(x)) \Rightarrow w'(x) = g'(g(x))g'(x)$. So $w'(1) = g'(g(1))g'(1) = g'(3)g'(1)$. To find $g'(3)$, note that g is linear from $(2,0)$ to $(5,2)$, so its slope is $\dfrac{2-0}{5-2} = \dfrac{2}{3}$. Thus, $g'(3)g'(1) = \left(\frac{2}{3}\right)(-3) = -2$.

67. The point $(3,2)$ is on the graph of f, so $f(3) = 2$. The tangent line at $(3,2)$ has slope $\dfrac{\Delta y}{\Delta x} = \dfrac{-4}{6} = -\dfrac{2}{3}$.

$g(x) = \sqrt{f(x)} \Rightarrow g'(x) = \frac{1}{2}[f(x)]^{-1/2} \cdot f'(x) \Rightarrow$

$g'(3) = \frac{1}{2}[f(3)]^{-1/2} \cdot f'(3) = \frac{1}{2}(2)^{-1/2}\left(-\frac{2}{3}\right) = -\dfrac{1}{3\sqrt{2}}$ or $-\frac{1}{6}\sqrt{2}$.

69. (a) $F(x) = f(e^x) \Rightarrow F'(x) = f'(e^x)\dfrac{d}{dx}(e^x) = f'(e^x)e^x$

(b) $G(x) = e^{f(x)} \Rightarrow G'(x) = e^{f(x)}\dfrac{d}{dx}f(x) = e^{f(x)}f'(x)$

71. $r(x) = f(g(h(x))) \Rightarrow r'(x) = f'(g(h(x))) \cdot g'(h(x)) \cdot h'(x)$, so

$r'(1) = f'(g(h(1))) \cdot g'(h(1)) \cdot h'(1) = f'(g(2)) \cdot g'(2) \cdot 4 = f'(3) \cdot 5 \cdot 4 = 6 \cdot 5 \cdot 4 = 120$

73. $F(x) = f(3f(4f(x))) \Rightarrow$

$$F'(x) = f'(3f(4f(x))) \cdot \frac{d}{dx}(3f(4f(x))) = f'(3f(4f(x))) \cdot 3f'(4f(x)) \cdot \frac{d}{dx}(4f(x))$$

$$= f'(3f(4f(x))) \cdot 3f'(4f(x)) \cdot 4f'(x), \text{ so}$$

$$F'(0) = f'(3f(4f(0))) \cdot 3f'(4f(0)) \cdot 4f'(0) = f'(3f(4 \cdot 0)) \cdot 3f'(4 \cdot 0) \cdot 4 \cdot 2 = f'(3 \cdot 0) \cdot 3 \cdot 2 \cdot 4 \cdot 2 = 2 \cdot 3 \cdot 2 \cdot 4 \cdot 2 = 96.$$

75. $y = e^{2x}(A\cos 3x + B\sin 3x) \Rightarrow$

$$\begin{aligned} y' &= e^{2x}(-3A\sin 3x + 3B\cos 3x) + (A\cos 3x + B\sin 3x)\cdot 2e^{2x} \\ &= e^{2x}(-3A\sin 3x + 3B\cos 3x + 2A\cos 3x + 2B\sin 3x) \\ &= e^{2x}[(2A+3B)\cos 3x + (2B-3A)\sin 3x] \quad \Rightarrow \end{aligned}$$

$$\begin{aligned} y'' &= e^{2x}[-3(2A+3B)\sin 3x + 3(2B-3A)\cos 3x] + [(2A+3B)\cos 3x + (2B-3A)\sin 3x]\cdot 2e^{2x} \\ &= e^{2x}\{[-3(2A+3B) + 2(2B-3A)]\sin 3x + [3(2B-3A) + 2(2A+3B)]\cos 3x\} \\ &= e^{2x}[(-12A-5B)\sin 3x + (-5A+12B)\cos 3x] \end{aligned}$$

Substitute the expressions for y, y', and y'' in $y'' - 4y' + 13y$ to get

$$\begin{aligned} y'' - 4y' + 13y &= e^{2x}[(-12A-5B)\sin 3x + (-5A+12B)\cos 3x] \\ &\quad - 4e^{2x}[(2A+3B)\cos 3x + (2B-3A)\sin 3x] + 13e^{2x}(A\cos 3x + B\sin 3x) \\ &= e^{2x}[(-12A - 5B - 8B + 12A + 13B)\sin 3x + (-5A + 12B - 8A - 12B + 13A)\cos 3x] \\ &= e^{2x}[(0)\sin 3x + (0)\cos 3x] = 0 \end{aligned}$$

Thus, the function y satisfies the differential equation $y'' - 4y' + 13y = 0$.

77. The use of $D, D^2, \ldots, D^n$ is just a derivative notation (see text page 157). In general, $Df(2x) = 2f'(2x)$, $D^2 f(2x) = 4f''(2x), \ldots, D^n f(2x) = 2^n f^{(n)}(2x)$. Since $f(x) = \cos x$ and $50 = 4(12) + 2$, we have $f^{(50)}(x) = f^{(2)}(x) = -\cos x$, so $D^{50}\cos 2x = -2^{50}\cos 2x$.

79. $s(t) = 10 + \frac{1}{4}\sin(10\pi t) \Rightarrow$ the velocity after t seconds is $v(t) = s'(t) = \frac{1}{4}\cos(10\pi t)(10\pi) = \frac{5\pi}{2}\cos(10\pi t)$ cm/s.

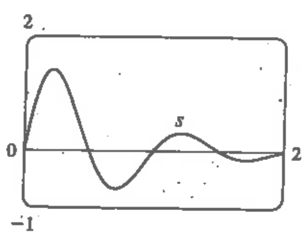
81. (a) $B(t) = 4.0 + 0.35\sin\dfrac{2\pi t}{5.4} \Rightarrow \dfrac{dB}{dt} = \left(0.35\cos\dfrac{2\pi t}{5.4}\right)\left(\dfrac{2\pi}{5.4}\right) = \dfrac{0.7\pi}{5.4}\cos\dfrac{2\pi t}{5.4} = \dfrac{7\pi}{54}\cos\dfrac{2\pi t}{5.4}$

(b) At $t = 1$, $\dfrac{dB}{dt} = \dfrac{7\pi}{54}\cos\dfrac{2\pi}{5.4} \approx 0.16$.

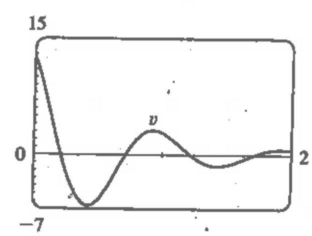
83. $s(t) = 2e^{-1.5t}\sin 2\pi t \Rightarrow$

$v(t) = s'(t) = 2[e^{-1.5t}(\cos 2\pi t)(2\pi) + (\sin 2\pi t)e^{-1.5t}(-1.5)] = 2e^{-1.5t}(2\pi\cos 2\pi t - 1.5\sin 2\pi t)$

Graph of position



Graph of velocity



85. By the Chain Rule, $a(t) = \dfrac{dv}{dt} = \dfrac{dv}{ds}\dfrac{ds}{dt} = \dfrac{dv}{ds}v(t) = v(t)\dfrac{dv}{ds}$. The derivative dv/dt is the rate of change of the velocity with respect to time (in other words, the acceleration) whereas the derivative dv/ds is the rate of change of the velocity with respect to the displacement.