

There appear to be vertical asymptotes of the curve $y = \tan(2 \sin x)$ at $x \approx \pm 0.90$ and $x \approx \pm 2.24$. To find the exact equations of these asymptotes, we note that the graph of the tangent function has vertical asymptotes at $x = \frac{\pi}{2} + \pi n$. Thus, we must have $2 \sin x = \frac{\pi}{2} + \pi n$, or equivalently, $\sin x = \frac{\pi}{4} + \frac{\pi}{2} n$. Since $-1 \leq \sin x \leq 1$, we must have $\sin x = \pm \frac{\pi}{4}$ and so $x = \pm \sin^{-1} \frac{\pi}{4}$ (corresponding to $x \approx \pm 0.90$). Just as 150° is the reference angle for 30° , $\pi - \sin^{-1} \frac{\pi}{4}$ is the reference angle for $\sin^{-1} \frac{\pi}{4}$. So $x = \pm(\pi - \sin^{-1} \frac{\pi}{4})$ are also equations of vertical asymptotes (corresponding to $x \approx \pm 2.24$).

2.3 Calculating Limits Using the Limit Laws

1. (a) $\lim_{x \rightarrow 2} [f(x) + 5g(x)] = \lim_{x \rightarrow 2} f(x) + \lim_{x \rightarrow 2} [5g(x)]$ [Limit Law 1]
 $= \lim_{x \rightarrow 2} f(x) + 5 \lim_{x \rightarrow 2} g(x)$ [Limit Law 3]
 $= 4 + 5(-2) = -6$

(b) $\lim_{x \rightarrow 2} [g(x)]^3 = \left[\lim_{x \rightarrow 2} g(x) \right]^3$ [Limit Law 6]
 $= (-2)^3 = -8$

(c) $\lim_{x \rightarrow 2} \sqrt{f(x)} = \sqrt{\lim_{x \rightarrow 2} f(x)}$ [Limit Law 11]
 $= \sqrt{4} = 2$

(d) $\lim_{x \rightarrow 2} \frac{3f(x)}{g(x)} = \frac{\lim_{x \rightarrow 2} [3f(x)]}{\lim_{x \rightarrow 2} g(x)}$ [Limit Law 5]
 $= \frac{3 \lim_{x \rightarrow 2} f(x)}{\lim_{x \rightarrow 2} g(x)}$ [Limit Law 3]
 $= \frac{3(4)}{-2} = -6$

(e) Because the limit of the denominator is 0, we can't use Limit Law 5. The given limit, $\lim_{x \rightarrow 2} \frac{g(x)}{h(x)}$, does not exist because the denominator approaches 0 while the numerator approaches a nonzero number.

(f) $\lim_{x \rightarrow 2} \frac{g(x)h(x)}{f(x)} = \frac{\lim_{x \rightarrow 2} [g(x)h(x)]}{\lim_{x \rightarrow 2} f(x)}$ [Limit Law 5]
 $= \frac{\lim_{x \rightarrow 2} g(x) \cdot \lim_{x \rightarrow 2} h(x)}{\lim_{x \rightarrow 2} f(x)}$ [Limit Law 4]
 $= \frac{-2 \cdot 0}{4} = 0$

3. $\lim_{x \rightarrow -2} (3x^4 + 2x^2 - x + 1) = \lim_{x \rightarrow -2} 3x^4 + \lim_{x \rightarrow -2} 2x^2 - \lim_{x \rightarrow -2} x + \lim_{x \rightarrow -2} 1$ [Limit Laws 1 and 2]
 $= 3 \lim_{x \rightarrow -2} x^4 + 2 \lim_{x \rightarrow -2} x^2 - \lim_{x \rightarrow -2} x + \lim_{x \rightarrow -2} 1$ [3]
 $= 3(-2)^4 + 2(-2)^2 - (-2) + (1)$ [9, 8, and 7]
 $= 48 + 8 + 2 + 1 = 59$

$$5. \lim_{t \rightarrow -2} \frac{t^4 - 2}{2t^2 - 3t + 2} = \frac{\lim_{t \rightarrow -2} (t^4 - 2)}{\lim_{t \rightarrow -2} (2t^2 - 3t + 2)} \quad \text{[Limit Law 5]}$$

$$= \frac{\lim_{t \rightarrow -2} t^4 - \lim_{t \rightarrow -2} 2}{2 \lim_{t \rightarrow -2} t^2 - 3 \lim_{t \rightarrow -2} t + \lim_{t \rightarrow -2} 2} \quad \text{[1, 2, and 3]}$$

$$= \frac{16 - 2}{2(4) - 3(-2) + 2} \quad \text{[9, 7, and 8]}$$

$$= \frac{14}{16} = \frac{7}{8}$$

$$7. \lim_{x \rightarrow 8} (1 + \sqrt[3]{x})(2 - 6x^2 + x^3) = \lim_{x \rightarrow 8} (1 + \sqrt[3]{x}) \cdot \lim_{x \rightarrow 8} (2 - 6x^2 + x^3) \quad \text{[Limit Law 4]}$$

$$= \left(\lim_{x \rightarrow 8} 1 + \lim_{x \rightarrow 8} \sqrt[3]{x} \right) \cdot \left(\lim_{x \rightarrow 8} 2 - 6 \lim_{x \rightarrow 8} x^2 + \lim_{x \rightarrow 8} x^3 \right) \quad \text{[1, 2, and 3]}$$

$$= (1 + \sqrt[3]{8}) \cdot (2 - 6 \cdot 8^2 + 8^3) \quad \text{[7, 10, 9]}$$

$$= (3)(130) = 390$$

$$9. \lim_{x \rightarrow 2} \sqrt{\frac{2x^2 + 1}{3x - 2}} = \sqrt{\lim_{x \rightarrow 2} \frac{2x^2 + 1}{3x - 2}} \quad \text{[Limit Law 11]}$$

$$= \sqrt{\frac{\lim_{x \rightarrow 2} (2x^2 + 1)}{\lim_{x \rightarrow 2} (3x - 2)}} \quad \text{[5]}$$

$$= \sqrt{\frac{2 \lim_{x \rightarrow 2} x^2 + \lim_{x \rightarrow 2} 1}{3 \lim_{x \rightarrow 2} x - \lim_{x \rightarrow 2} 2}} \quad \text{[1, 2, and 3]}$$

$$= \sqrt{\frac{2(2)^2 + 1}{3(2) - 2}} = \sqrt{\frac{9}{4}} = \frac{3}{2} \quad \text{[9, 8, and 7]}$$

$$11. \lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x - 2} = \lim_{x \rightarrow 2} \frac{(x + 3)(x - 2)}{x - 2} = \lim_{x \rightarrow 2} (x + 3) = 2 + 3 = 5$$

$$13. \lim_{x \rightarrow 2} \frac{x^2 - x + 6}{x - 2} \text{ does not exist since } x - 2 \rightarrow 0 \text{ but } x^2 - x + 6 \rightarrow 8 \text{ as } x \rightarrow 2.$$

$$15. \lim_{t \rightarrow -3} \frac{t^2 - 9}{2t^2 + 7t + 3} = \lim_{t \rightarrow -3} \frac{(t + 3)(t - 3)}{(2t + 1)(t + 3)} = \lim_{t \rightarrow -3} \frac{t - 3}{2t + 1} = \frac{-3 - 3}{2(-3) + 1} = \frac{-6}{-5} = \frac{6}{5}$$

$$17. \lim_{h \rightarrow 0} \frac{(-5 + h)^2 - 25}{h} = \lim_{h \rightarrow 0} \frac{(25 - 10h + h^2) - 25}{h} = \lim_{h \rightarrow 0} \frac{-10h + h^2}{h} = \lim_{h \rightarrow 0} \frac{h(-10 + h)}{h} = \lim_{h \rightarrow 0} (-10 + h) = -10$$

19. By the formula for the sum of cubes, we have

$$\lim_{x \rightarrow -2} \frac{x + 2}{x^3 + 8} = \lim_{x \rightarrow -2} \frac{x + 2}{(x + 2)(x^2 - 2x + 4)} = \lim_{x \rightarrow -2} \frac{1}{x^2 - 2x + 4} = \frac{1}{4 + 4 + 4} = \frac{1}{12}$$

$$\begin{aligned}
 21. \lim_{h \rightarrow 0} \frac{\sqrt{9+h}-3}{h} &= \lim_{h \rightarrow 0} \frac{\sqrt{9+h}-3}{h} \cdot \frac{\sqrt{9+h}+3}{\sqrt{9+h}+3} = \lim_{h \rightarrow 0} \frac{(\sqrt{9+h})^2 - 3^2}{h(\sqrt{9+h}+3)} = \lim_{h \rightarrow 0} \frac{(9+h)-9}{h(\sqrt{9+h}+3)} \\
 &= \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{9+h}+3)} = \lim_{h \rightarrow 0} \frac{1}{\sqrt{9+h}+3} = \frac{1}{3+3} = \frac{1}{6}
 \end{aligned}$$

$$23. \lim_{x \rightarrow -4} \frac{\frac{1}{4} + \frac{1}{x}}{4+x} = \lim_{x \rightarrow -4} \frac{\frac{x+4}{4x}}{4+x} = \lim_{x \rightarrow -4} \frac{x+4}{4x(4+x)} = \lim_{x \rightarrow -4} \frac{1}{4x} = \frac{1}{4(-4)} = -\frac{1}{16}$$

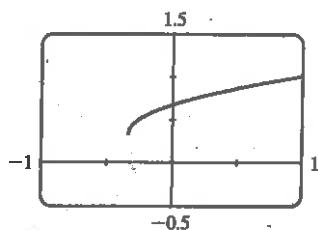
$$\begin{aligned}
 25. \lim_{t \rightarrow 0} \frac{\sqrt{1+t}-\sqrt{1-t}}{t} &= \lim_{t \rightarrow 0} \frac{\sqrt{1+t}-\sqrt{1-t}}{t} \cdot \frac{\sqrt{1+t}+\sqrt{1-t}}{\sqrt{1+t}+\sqrt{1-t}} = \lim_{t \rightarrow 0} \frac{(\sqrt{1+t})^2 - (\sqrt{1-t})^2}{t(\sqrt{1+t}+\sqrt{1-t})} \\
 &= \lim_{t \rightarrow 0} \frac{(1+t) - (1-t)}{t(\sqrt{1+t}+\sqrt{1-t})} = \lim_{t \rightarrow 0} \frac{2t}{t(\sqrt{1+t}+\sqrt{1-t})} = \lim_{t \rightarrow 0} \frac{2}{\sqrt{1+t}+\sqrt{1-t}} \\
 &= \frac{2}{\sqrt{1+0}+\sqrt{1-0}} = \frac{2}{2} = 1
 \end{aligned}$$

$$\begin{aligned}
 27. \lim_{x \rightarrow 16} \frac{4-\sqrt{x}}{16x-x^2} &= \lim_{x \rightarrow 16} \frac{(4-\sqrt{x})(4+\sqrt{x})}{(16x-x^2)(4+\sqrt{x})} = \lim_{x \rightarrow 16} \frac{16-x}{x(16-x)(4+\sqrt{x})} \\
 &= \lim_{x \rightarrow 16} \frac{1}{x(4+\sqrt{x})} = \frac{1}{16(4+\sqrt{16})} = \frac{1}{16(8)} = \frac{1}{128}
 \end{aligned}$$

$$\begin{aligned}
 29. \lim_{t \rightarrow 0} \left(\frac{1}{t\sqrt{1+t}} - \frac{1}{t} \right) &= \lim_{t \rightarrow 0} \frac{1-\sqrt{1+t}}{t\sqrt{1+t}} = \lim_{t \rightarrow 0} \frac{(1-\sqrt{1+t})(1+\sqrt{1+t})}{t\sqrt{1+t}(1+\sqrt{1+t})} = \lim_{t \rightarrow 0} \frac{-t}{t\sqrt{1+t}(1+\sqrt{1+t})} \\
 &= \lim_{t \rightarrow 0} \frac{-1}{\sqrt{1+t}(1+\sqrt{1+t})} = \frac{-1}{\sqrt{1+0}(1+\sqrt{1+0})} = -\frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 31. \lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h} &= \lim_{h \rightarrow 0} \frac{(x^3 + 3x^2h + 3xh^2 + h^3) - x^3}{h} = \lim_{h \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3}{h} \\
 &= \lim_{h \rightarrow 0} \frac{h(3x^2 + 3xh + h^2)}{h} = \lim_{h \rightarrow 0} (3x^2 + 3xh + h^2) = 3x^2
 \end{aligned}$$

33. (a)



$$\lim_{x \rightarrow 0} \frac{x}{\sqrt{1+3x}-1} \approx \frac{2}{3}$$

(b)

x	$f(x)$
-0.001	0.6661663
-0.0001	0.6666167
-0.00001	0.6666617
-0.000001	0.6666662
0.000001	0.6666672
0.00001	0.6666717
0.0001	0.6667167
0.001	0.6671663

The limit appears to be $\frac{2}{3}$.

$$\begin{aligned}
 \text{(c) } \lim_{x \rightarrow 0} \left(\frac{x}{\sqrt{1+3x}-1} \cdot \frac{\sqrt{1+3x}+1}{\sqrt{1+3x}+1} \right) &= \lim_{x \rightarrow 0} \frac{x(\sqrt{1+3x}+1)}{(1+3x)-1} = \lim_{x \rightarrow 0} \frac{x(\sqrt{1+3x}+1)}{3x} \\
 &= \frac{1}{3} \lim_{x \rightarrow 0} (\sqrt{1+3x}+1) && \text{[Limit Law 3]} \\
 &= \frac{1}{3} \left[\lim_{x \rightarrow 0} (\sqrt{1+3x}) + \lim_{x \rightarrow 0} 1 \right] && \text{[1 and 11]} \\
 &= \frac{1}{3} \left(\sqrt{\lim_{x \rightarrow 0} 1 + 3 \lim_{x \rightarrow 0} x} + 1 \right) && \text{[1, 3, and 7]} \\
 &= \frac{1}{3} (\sqrt{1+3 \cdot 0} + 1) && \text{[7 and 8]} \\
 &= \frac{1}{3} (1+1) = \frac{2}{3}
 \end{aligned}$$

35. Let $f(x) = -x^2$, $g(x) = x^2 \cos 20\pi x$ and $h(x) = x^2$. Then

$$-1 \leq \cos 20\pi x \leq 1 \Rightarrow -x^2 \leq x^2 \cos 20\pi x \leq x^2 \Rightarrow f(x) \leq g(x) \leq h(x).$$

So since $\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} h(x) = 0$, by the Squeeze Theorem we have

$$\lim_{x \rightarrow 0} g(x) = 0.$$

37. We have $\lim_{x \rightarrow 4} (4x - 9) = 4(4) - 9 = 7$ and $\lim_{x \rightarrow 4} (x^2 - 4x + 7) = 4^2 - 4(4) + 7 = 7$. Since $4x - 9 \leq f(x) \leq x^2 - 4x + 7$

for $x \geq 0$, $\lim_{x \rightarrow 4} f(x) = 7$ by the Squeeze Theorem.

39. $-1 \leq \cos(2/x) \leq 1 \Rightarrow -x^4 \leq x^4 \cos(2/x) \leq x^4$. Since $\lim_{x \rightarrow 0} (-x^4) = 0$ and $\lim_{x \rightarrow 0} x^4 = 0$, we have

$\lim_{x \rightarrow 0} [x^4 \cos(2/x)] = 0$ by the Squeeze Theorem.

$$41. |x-3| = \begin{cases} x-3 & \text{if } x-3 \geq 0 \\ -(x-3) & \text{if } x-3 < 0 \end{cases} = \begin{cases} x-3 & \text{if } x \geq 3 \\ 3-x & \text{if } x < 3 \end{cases}$$

Thus, $\lim_{x \rightarrow 3^+} (2x + |x-3|) = \lim_{x \rightarrow 3^+} (2x + x - 3) = \lim_{x \rightarrow 3^+} (3x - 3) = 3(3) - 3 = 6$ and

$\lim_{x \rightarrow 3^-} (2x + |x-3|) = \lim_{x \rightarrow 3^-} (2x + 3 - x) = \lim_{x \rightarrow 3^-} (x + 3) = 3 + 3 = 6$. Since the left and right limits are equal,

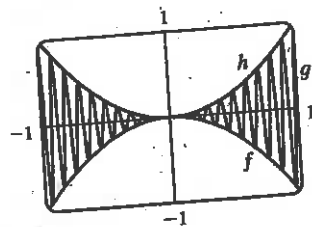
$$\lim_{x \rightarrow 3} (2x + |x-3|) = 6.$$

$$43. |2x^3 - x^2| = |x^2(2x-1)| = |x^2| \cdot |2x-1| = x^2 |2x-1|$$

$$|2x-1| = \begin{cases} 2x-1 & \text{if } 2x-1 \geq 0 \\ -(2x-1) & \text{if } 2x-1 < 0 \end{cases} = \begin{cases} 2x-1 & \text{if } x \geq 0.5 \\ -(2x-1) & \text{if } x < 0.5 \end{cases}$$

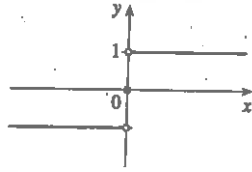
So $|2x^3 - x^2| = x^2[-(2x-1)]$ for $x < 0.5$.

$$\text{Thus, } \lim_{x \rightarrow 0.5^-} \frac{2x-1}{|2x^3-x^2|} = \lim_{x \rightarrow 0.5^-} \frac{2x-1}{x^2[-(2x-1)]} = \lim_{x \rightarrow 0.5^-} \frac{-1}{x^2} = \frac{-1}{(0.5)^2} = \frac{-1}{0.25} = -4.$$



45. Since $|x| = -x$ for $x < 0$, we have $\lim_{x \rightarrow 0^-} \left(\frac{1}{x} - \frac{1}{|x|} \right) = \lim_{x \rightarrow 0^-} \left(\frac{1}{x} - \frac{1}{-x} \right) = \lim_{x \rightarrow 0^-} \frac{2}{x}$, which does not exist since the denominator approaches 0 and the numerator does not.

47. (a)



(b) (i) Since $\operatorname{sgn} x = 1$ for $x > 0$, $\lim_{x \rightarrow 0^+} \operatorname{sgn} x = \lim_{x \rightarrow 0^+} 1 = 1$.

(ii) Since $\operatorname{sgn} x = -1$ for $x < 0$, $\lim_{x \rightarrow 0^-} \operatorname{sgn} x = \lim_{x \rightarrow 0^-} -1 = -1$.

(iii) Since $\lim_{x \rightarrow 0^-} \operatorname{sgn} x \neq \lim_{x \rightarrow 0^+} \operatorname{sgn} x$, $\lim_{x \rightarrow 0} \operatorname{sgn} x$ does not exist.

(iv) Since $|\operatorname{sgn} x| = 1$ for $x \neq 0$, $\lim_{x \rightarrow 0} |\operatorname{sgn} x| = \lim_{x \rightarrow 0} 1 = 1$.

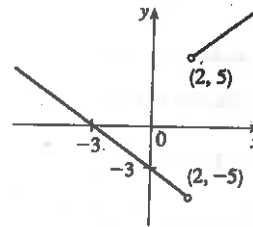
49. (a) (i) $\lim_{x \rightarrow 2^+} g(x) = \lim_{x \rightarrow 2^+} \frac{x^2 + x - 6}{|x - 2|} = \lim_{x \rightarrow 2^+} \frac{(x + 3)(x - 2)}{|x - 2|}$
 $= \lim_{x \rightarrow 2^+} \frac{(x + 3)(x - 2)}{x - 2}$ [since $x - 2 > 0$ if $x \rightarrow 2^+$]
 $= \lim_{x \rightarrow 2^+} (x + 3) = 5$

(ii) The solution is similar to the solution in part (i), but now $|x - 2| = 2 - x$ since $x - 2 < 0$ if $x \rightarrow 2^-$.

Thus, $\lim_{x \rightarrow 2^-} g(x) = \lim_{x \rightarrow 2^-} -(x + 3) = -5$.

(b) Since the right-hand and left-hand limits of g at $x = 2$ are not equal, $\lim_{x \rightarrow 2} g(x)$ does not exist.

(c)



51. (a) (i) $[x] = -2$ for $-2 \leq x < -1$, so $\lim_{x \rightarrow -2^+} [x] = \lim_{x \rightarrow -2^+} (-2) = -2$

(ii) $[x] = -3$ for $-3 \leq x < -2$, so $\lim_{x \rightarrow -2^-} [x] = \lim_{x \rightarrow -2^-} (-3) = -3$.

The right and left limits are different, so $\lim_{x \rightarrow -2} [x]$ does not exist.

(iii) $[x] = -3$ for $-3 \leq x < -2$, so $\lim_{x \rightarrow -2.4} [x] = \lim_{x \rightarrow -2.4} (-3) = -3$.

(b) (i) $[x] = n - 1$ for $n - 1 \leq x < n$, so $\lim_{x \rightarrow n^-} [x] = \lim_{x \rightarrow n^-} (n - 1) = n - 1$.

(ii) $[x] = n$ for $n \leq x < n + 1$, so $\lim_{x \rightarrow n^+} [x] = \lim_{x \rightarrow n^+} n = n$.

(c) $\lim_{x \rightarrow a} [x]$ exists $\Leftrightarrow a$ is not an integer.

53. The graph of $f(x) = [x] + [-x]$ is the same as the graph of $g(x) = -1$ with holes at each integer, since $f(a) = 0$ for any integer a . Thus, $\lim_{x \rightarrow 2^-} f(x) = -1$ and $\lim_{x \rightarrow 2^+} f(x) = -1$, so $\lim_{x \rightarrow 2} f(x) = -1$. However, $f(2) = [2] + [-2] = 2 + (-2) = 0$, so $\lim_{x \rightarrow 2} f(x) \neq f(2)$.

55. Since $p(x)$ is a polynomial, $p(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$. Thus, by the Limit Laws,

$$\begin{aligned} \lim_{x \rightarrow a} p(x) &= \lim_{x \rightarrow a} (a_0 + a_1x + a_2x^2 + \dots + a_nx^n) = a_0 + a_1 \lim_{x \rightarrow a} x + a_2 \lim_{x \rightarrow a} x^2 + \dots + a_n \lim_{x \rightarrow a} x^n \\ &= a_0 + a_1a + a_2a^2 + \dots + a_na^n = p(a). \end{aligned}$$

Thus, for any polynomial p , $\lim_{x \rightarrow a} p(x) = p(a)$.

57. $\lim_{x \rightarrow 1} [f(x) - 8] = \lim_{x \rightarrow 1} \left[\frac{f(x) - 8}{x - 1} \cdot (x - 1) \right] = \lim_{x \rightarrow 1} \frac{f(x) - 8}{x - 1} \cdot \lim_{x \rightarrow 1} (x - 1) = 10 \cdot 0 = 0.$

Thus, $\lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} \{ [f(x) - 8] + 8 \} = \lim_{x \rightarrow 1} [f(x) - 8] + \lim_{x \rightarrow 1} 8 = 0 + 8 = 8.$

Note: The value of $\lim_{x \rightarrow 1} \frac{f(x) - 8}{x - 1}$ does not affect the answer since it's multiplied by 0. What's important is that

$\lim_{x \rightarrow 1} \frac{f(x) - 8}{x - 1}$ exists.

59. Observe that $0 \leq f(x) \leq x^2$ for all x , and $\lim_{x \rightarrow 0} 0 = 0 = \lim_{x \rightarrow 0} x^2$. So, by the Squeeze Theorem, $\lim_{x \rightarrow 0} f(x) = 0$.

61. Let $f(x) = H(x)$ and $g(x) = 1 - H(x)$, where H is the Heaviside function defined in Exercise 1.3.57.

Thus, either f or g is 0 for any value of x . Then $\lim_{x \rightarrow 0} f(x)$ and $\lim_{x \rightarrow 0} g(x)$ do not exist, but $\lim_{x \rightarrow 0} [f(x)g(x)] = \lim_{x \rightarrow 0} 0 = 0$.

63. Since the denominator approaches 0 as $x \rightarrow -2$, the limit will exist only if the numerator also approaches

0 as $x \rightarrow -2$. In order for this to happen, we need $\lim_{x \rightarrow -2} (3x^2 + ax + a + 3) = 0 \Leftrightarrow$

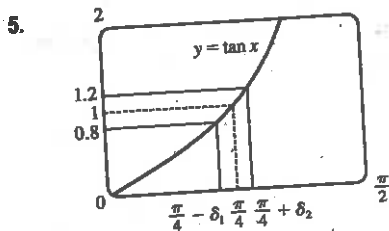
$3(-2)^2 + a(-2) + a + 3 = 0 \Leftrightarrow 12 - 2a + a + 3 = 0 \Leftrightarrow a = 15$. With $a = 15$, the limit becomes

$$\lim_{x \rightarrow -2} \frac{3x^2 + 15x + 18}{x^2 + x - 2} = \lim_{x \rightarrow -2} \frac{3(x+2)(x+3)}{(x-1)(x+2)} = \lim_{x \rightarrow -2} \frac{3(x+3)}{x-1} = \frac{3(-2+3)}{-2-1} = \frac{3}{-3} = -1.$$

2.4 The Precise Definition of a Limit

1. If $|f(x) - 1| < 0.2$, then $-0.2 < f(x) - 1 < 0.2 \Rightarrow 0.8 < f(x) < 1.2$. From the graph, we see that the last inequality is true if $0.7 < x < 1.1$, so we can choose $\delta = \min \{1 - 0.7, 1.1 - 1\} = \min \{0.3, 0.1\} = 0.1$ (or any smaller positive number).

3. The leftmost question mark is the solution of $\sqrt{x} = 1.6$ and the rightmost, $\sqrt{x} = 2.4$. So the values are $1.6^2 = 2.56$ and $2.4^2 = 5.76$. On the left side, we need $|x - 4| < |2.56 - 4| = 1.44$. On the right side, we need $|x - 4| < |5.76 - 4| = 1.76$. To satisfy both conditions, we need the more restrictive condition to hold—namely, $|x - 4| < 1.44$. Thus, we can choose $\delta = 1.44$, or any smaller positive number.



From the graph, we find that $y = \tan x = 0.8$ when $x \approx 0.675$, so $\frac{\pi}{4} - \delta_1 \approx 0.675 \Rightarrow \delta_1 \approx \frac{\pi}{4} - 0.675 \approx 0.1106$. Also, $y = \tan x = 1.2$ when $x \approx 0.876$, so $\frac{\pi}{4} + \delta_2 \approx 0.876 \Rightarrow \delta_2 = 0.876 - \frac{\pi}{4} \approx 0.0906$. Thus, we choose $\delta = 0.0906$ (or any smaller positive number) since this is the smaller of δ_1 and δ_2 .