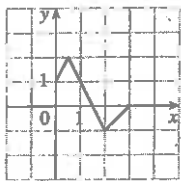


- (d) It is *not* reasonable to use the model to predict the winning height at the 2100 Olympics since 2100 is too far from the 1896–2004 range on which the model is based.
25. If x is the original distance from the source, then the illumination is $f(x) = kx^{-2} = k/x^2$. Moving halfway to the lamp gives us an illumination of $f(\frac{1}{2}x) = k(\frac{1}{2}x)^{-2} = k(2/x)^2 = 4(k/x^2)$, so the light is 4 times as bright.
27. (a) Using a computing device, we obtain a power function $N = cA^b$, where $c \approx 2.3356$ and $b \approx 0.3072$.
- (b) If $A = 754$, then $N = cA^b \approx 17.88$, so you would expect to find 18 species of reptiles and amphibians on Dominica.

1.3 New Functions from Old Functions

- If the graph of f is shifted 3 units upward, its equation becomes $y = f(x) + 3$.
 - If the graph of f is shifted 3 units downward, its equation becomes $y = f(x) - 3$.
 - If the graph of f is shifted 3 units to the right, its equation becomes $y = f(x - 3)$.
 - If the graph of f is shifted 3 units to the left, its equation becomes $y = f(x + 3)$.
 - If the graph of f is reflected about the x -axis, its equation becomes $y = -f(x)$.
 - If the graph of f is reflected about the y -axis, its equation becomes $y = f(-x)$.
 - If the graph of f is stretched vertically by a factor of 3, its equation becomes $y = 3f(x)$.
 - If the graph of f is shrunk vertically by a factor of 3, its equation becomes $y = \frac{1}{3}f(x)$.
- (graph 3) The graph of f is shifted 4 units to the right and has equation $y = f(x - 4)$.
 - (graph 1) The graph of f is shifted 3 units upward and has equation $y = f(x) + 3$.
 - (graph 4) The graph of f is shrunk vertically by a factor of 3 and has equation $y = \frac{1}{3}f(x)$.
 - (graph 5) The graph of f is shifted 4 units to the left and reflected about the x -axis. Its equation is $y = -f(x + 4)$.
 - (graph 2) The graph of f is shifted 6 units to the left and stretched vertically by a factor of 2. Its equation is $y = 2f(x + 6)$.
- To graph $y = f(2x)$ we shrink the graph of f horizontally by a factor of 2.
 - To graph $y = f(\frac{1}{2}x)$ we stretch the graph of f horizontally by a factor of 2.



The point $(4, -1)$ on the graph of f corresponds to the point $(\frac{1}{2} \cdot 4, -1) = (2, -1)$.



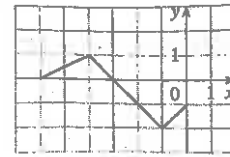
The point $(4, -1)$ on the graph of f corresponds to the point $(2 \cdot 4, -1) = (8, -1)$.

(c) To graph $y = f(-x)$ we reflect the graph of f about the y -axis.



The point $(4, -1)$ on the graph of f corresponds to the point $(-1 \cdot 4, -1) = (-4, -1)$.

(d) To graph $y = -f(-x)$ we reflect the graph of f about the y -axis, then about the x -axis.



The point $(4, -1)$ on the graph of f corresponds to the point $(-1 \cdot 4, -1 \cdot -1) = (-4, 1)$.

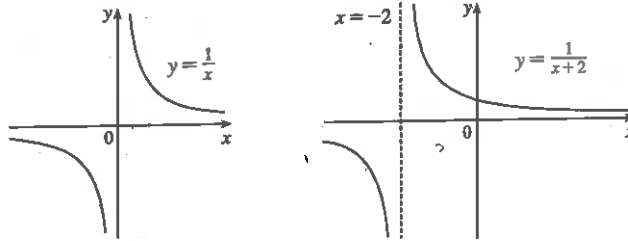
7. The graph of $y = f(x) = \sqrt{3x - x^2}$ has been shifted 4 units to the left, reflected about the x -axis, and shifted downward 1 unit. Thus, a function describing the graph is

$$y = \underbrace{-1}_{\text{reflect about } x\text{-axis}} \cdot \underbrace{f(x+4)}_{\text{shift 4 units left}} \underbrace{- 1}_{\text{shift 1 unit left}}$$

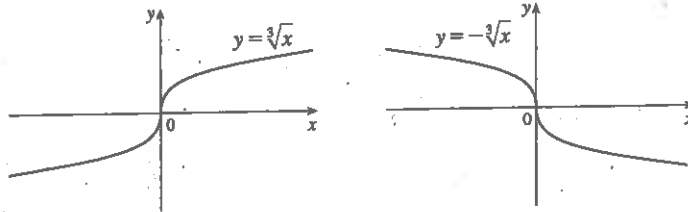
This function can be written as

$$y = -f(x+4) - 1 = -\sqrt{3(x+4) - (x+4)^2} - 1 = -\sqrt{3x+12 - (x^2+8x+16)} - 1 = -\sqrt{-x^2-5x-4} - 1$$

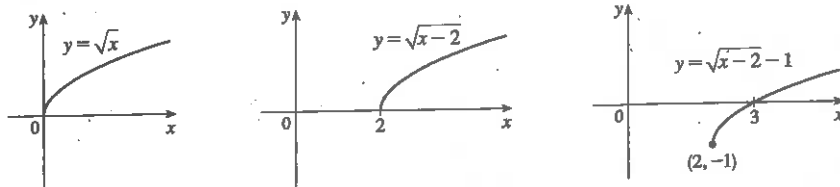
9. $y = \frac{1}{x+2}$: Start with the graph of the reciprocal function $y = 1/x$ and shift 2 units to the left.



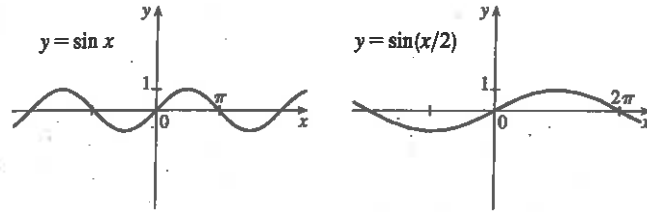
11. $y = -\sqrt[3]{x}$: Start with the graph of $y = \sqrt[3]{x}$ and reflect about the x -axis.



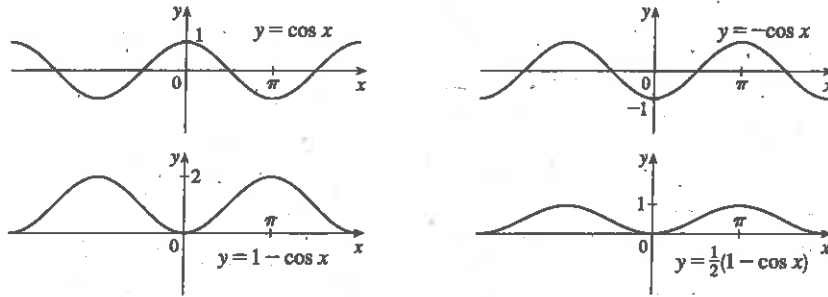
13. $y = \sqrt{x-2} - 1$: Start with the graph of $y = \sqrt{x}$, shift 2 units to the right, and then shift 1 unit downward.



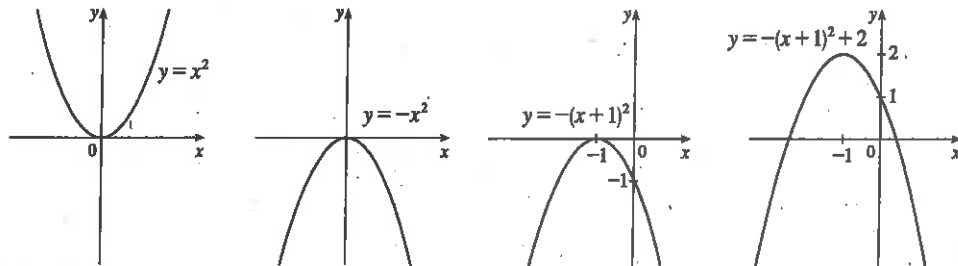
15. $y = \sin(x/2)$: Start with the graph of $y = \sin x$ and stretch horizontally by a factor of 2.



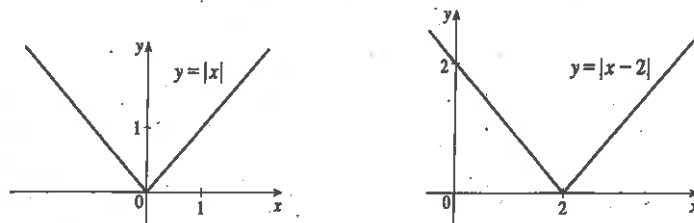
17. $y = \frac{1}{2}(1 - \cos x)$: Start with the graph of $y = \cos x$, reflect about the x -axis, shift 1 unit upward, and then shrink vertically by a factor of 2.



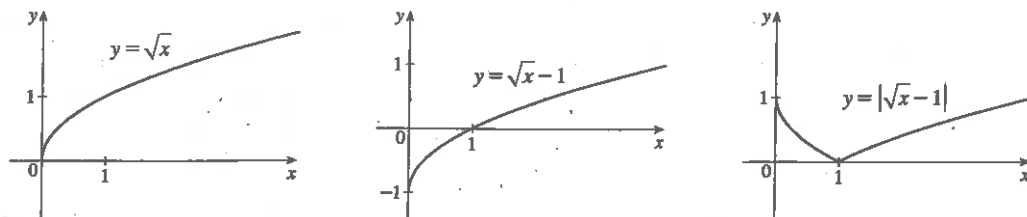
19. $y = 1 - 2x - x^2 = -(x^2 + 2x) + 1 = -(x^2 + 2x + 1) + 2 = -(x + 1)^2 + 2$: Start with the graph of $y = x^2$, reflect about the x -axis, shift 1 unit to the left, and then shift 2 units upward.



21. $y = |x - 2|$: Start with the graph of $y = |x|$ and shift 2 units to the right.



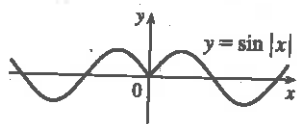
23. $y = |\sqrt{x} - 1|$: Start with the graph of $y = \sqrt{x}$, shift it 1 unit downward, and then reflect the portion of the graph below the x -axis about the x -axis.



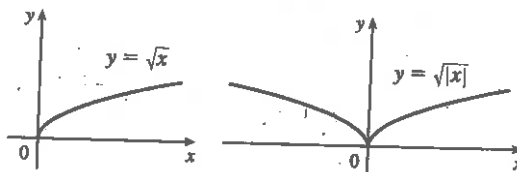
25. This is just like the solution to Example 4 except the amplitude of the curve (the 30°N curve in Figure 9 on June 21) is $14 - 12 = 2$. So the function is $L(t) = 12 + 2 \sin\left[\frac{2\pi}{365}(t - 80)\right]$. March 31 is the 90th day of the year, so the model gives $L(90) \approx 12.34$ h. The daylight time (6:13 AM to 6:39 PM) is 12 hours and 26 minutes, or 12.43 h. The model value differs from the actual value by $\frac{12.43 - 12.34}{12.43} \approx 0.007$, less than 1%.

27. (a) To obtain $y = f(|x|)$, the portion of the graph of $y = f(x)$ to the right of the y -axis is reflected about the y -axis.

(b) $y = \sin|x|$



(c) $y = \sqrt{|x|}$



29. $f(x) = x^3 + 2x^2$; $g(x) = 3x^2 - 1$. $D = \mathbb{R}$ for both f and g .

(a) $(f + g)(x) = (x^3 + 2x^2) + (3x^2 - 1) = x^3 + 5x^2 - 1$, $D = \mathbb{R}$.

(b) $(f - g)(x) = (x^3 + 2x^2) - (3x^2 - 1) = x^3 - x^2 + 1$, $D = \mathbb{R}$.

(c) $(fg)(x) = (x^3 + 2x^2)(3x^2 - 1) = 3x^5 + 6x^4 - x^3 - 2x^2$, $D = \mathbb{R}$.

(d) $\left(\frac{f}{g}\right)(x) = \frac{x^3 + 2x^2}{3x^2 - 1}$, $D = \left\{x \mid x \neq \pm \frac{1}{\sqrt{3}}\right\}$ since $3x^2 - 1 \neq 0$.

31. $f(x) = x^2 - 1$, $D = \mathbb{R}$; $g(x) = 2x + 1$, $D = \mathbb{R}$.

(a) $(f \circ g)(x) = f(g(x)) = f(2x + 1) = (2x + 1)^2 - 1 = (4x^2 + 4x + 1) - 1 = 4x^2 + 4x$, $D = \mathbb{R}$.

(b) $(g \circ f)(x) = g(f(x)) = g(x^2 - 1) = 2(x^2 - 1) + 1 = (2x^2 - 2) + 1 = 2x^2 - 1$, $D = \mathbb{R}$.

(c) $(f \circ f)(x) = f(f(x)) = f(x^2 - 1) = (x^2 - 1)^2 - 1 = (x^4 - 2x^2 + 1) - 1 = x^4 - 2x^2$, $D = \mathbb{R}$.

(d) $(g \circ g)(x) = g(g(x)) = g(2x + 1) = 2(2x + 1) + 1 = (4x + 2) + 1 = 4x + 3$, $D = \mathbb{R}$.

33. $f(x) = 1 - 3x$; $g(x) = \cos x$. $D = \mathbb{R}$ for both f and g , and hence for their composites.

(a) $(f \circ g)(x) = f(g(x)) = f(\cos x) = 1 - 3 \cos x$.

(b) $(g \circ f)(x) = g(f(x)) = g(1 - 3x) = \cos(1 - 3x)$.

(c) $(f \circ f)(x) = f(f(x)) = f(1 - 3x) = 1 - 3(1 - 3x) = 1 - 3 + 9x = 9x - 2$.

(d) $(g \circ g)(x) = g(g(x)) = g(\cos x) = \cos(\cos x)$ [Note that this is *not* $\cos x \cdot \cos x$.]

47. Let $h(x) = \sqrt{x}$, $g(x) = x - 1$, and $f(x) = \sqrt{x}$. Then

$$(f \circ g \circ h)(x) = f(g(h(x))) = f(g(\sqrt{x})) = f(\sqrt{x} - 1) = \sqrt{\sqrt{x} - 1} = R(x).$$

49. Let $h(x) = \sqrt{x}$, $g(x) = \sec x$, and $f(x) = x^4$. Then

$$(f \circ g \circ h)(x) = f(g(h(x))) = f(g(\sqrt{x})) = f(\sec \sqrt{x}) = (\sec \sqrt{x})^4 = \sec^4(\sqrt{x}) = H(x).$$

51. (a) $g(2) = 5$, because the point $(2, 5)$ is on the graph of g . Thus, $f(g(2)) = f(5) = 4$, because the point $(5, 4)$ is on the graph of f .

(b) $g(f(0)) = g(0) = 3$

(c) $(f \circ g)(0) = f(g(0)) = f(3) = 0$

(d) $(g \circ f)(6) = g(f(6)) = g(6)$. This value is not defined, because there is no point on the graph of g that has x -coordinate 6.

(e) $(g \circ g)(-2) = g(g(-2)) = g(1) = 4$

(f) $(f \circ f)(4) = f(f(4)) = f(2) = -2$

53. (a) Using the relationship $\text{distance} = \text{rate} \cdot \text{time}$ with the radius r as the distance, we have $r(t) = 60t$.

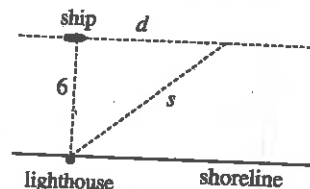
(b) $A = \pi r^2 \Rightarrow (A \circ r)(t) = A(r(t)) = \pi(60t)^2 = 3600\pi t^2$. This formula gives us the extent of the rippled area (in cm^2) at any time t .

55. (a) From the figure, we have a right triangle with legs 6 and d , and hypotenuse s .

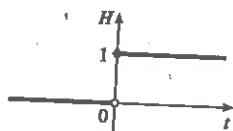
By the Pythagorean Theorem, $d^2 + 6^2 = s^2 \Rightarrow s = f(d) = \sqrt{d^2 + 36}$.

(b) Using $d = rt$, we get $d = (30 \text{ km/h})(t \text{ hours}) = 30t$ (in km). Thus,
 $d = g(t) = 30t$.

(c) $(f \circ g)(t) = f(g(t)) = f(30t) = \sqrt{(30t)^2 + 36} = \sqrt{900t^2 + 36}$. This function represents the distance between the lighthouse and the ship as a function of the time elapsed since noon.

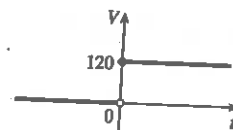


57. (a)



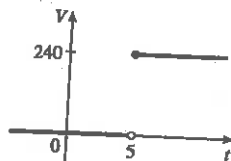
$$H(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1 & \text{if } t \geq 0 \end{cases}$$

(b)



$$V(t) = \begin{cases} 0 & \text{if } t < 0 \\ 120 & \text{if } t \geq 0 \end{cases} \quad \text{so } V(t) = 120H(t).$$

(c)



Starting with the formula in part (b), we replace 120 with 240 to reflect the different voltage. Also, because we are starting 5 units to the right of $t = 0$, we replace t with $t - 5$. Thus, the formula is $V(t) = 240H(t - 5)$.

59. If $f(x) = m_1x + b_1$ and $g(x) = m_2x + b_2$, then

$$(f \circ g)(x) = f(g(x)) = f(m_2x + b_2) = m_1(m_2x + b_2) + b_1 = m_1m_2x + m_1b_2 + b_1.$$

So $f \circ g$ is a linear function with slope m_1m_2 .

61. (a) By examining the variable terms in g and h , we deduce that we must square g to get the terms $4x^2$ and $4x$ in h . If we let

$$f(x) = x^2 + c, \text{ then } (f \circ g)(x) = f(g(x)) = f(2x + 1) = (2x + 1)^2 + c = 4x^2 + 4x + (1 + c). \text{ Since}$$

$$h(x) = 4x^2 + 4x + 7, \text{ we must have } 1 + c = 7. \text{ So } c = 6 \text{ and } f(x) = x^2 + 6.$$

(b) We need a function g so that $f(g(x)) = 3(g(x)) + 5 = h(x)$. But

$$h(x) = 3x^2 + 3x + 2 = 3(x^2 + x) + 2 = 3(x^2 + x - 1) + 5, \text{ so we see that } g(x) = x^2 + x - 1.$$

63. We need to examine $h(-x)$.

$$h(-x) = (f \circ g)(-x) = f(g(-x)) = f(g(x)) \quad [\text{because } g \text{ is even}] = h(x)$$

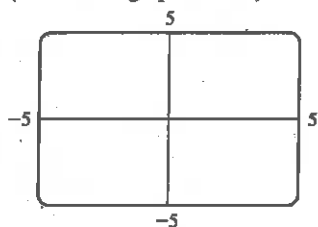
Because $h(-x) = h(x)$, h is an even function.

1.4 Graphing Calculators and Computers

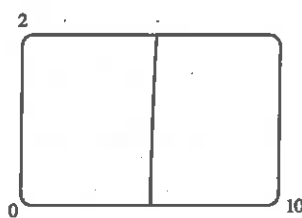
1. $f(x) = \sqrt{x^3 - 5x^2}$

(a) $[-5, 5]$ by $[-5, 5]$

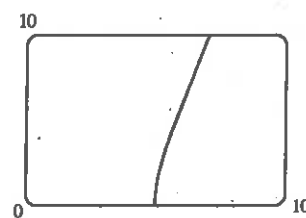
(There is no graph shown.)



(b) $[0, 10]$ by $[0, 2]$



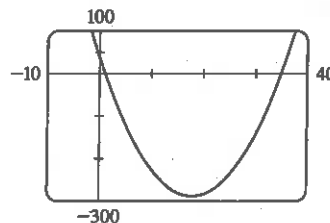
(c) $[0, 10]$ by $[0, 10]$



The most appropriate graph is produced in viewing rectangle (c).

3. Since the graph of $f(x) = x^2 - 36x + 32$ is a parabola opening upward, an appropriate viewing rectangle should include the minimum point.

Completing the square, we get $f(x) = (x - 18)^2 - 292$, and so the minimum point is $(18, -292)$.



5. $50 - 0.2x \geq 0 \Rightarrow 50 \geq 0.2x \Rightarrow x \leq 250$, so the domain of the root function $f(x) = \sqrt{50 - 0.2x}$ is $(-\infty, 250]$.

