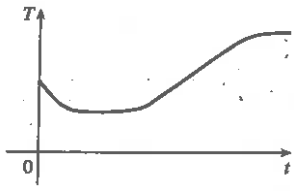
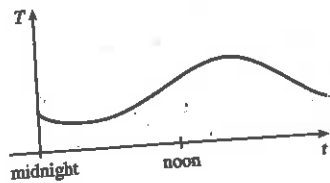


1 □ FUNCTIONS AND MODELS

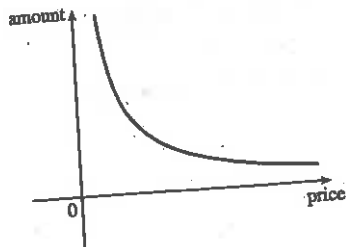
1.1 Four Ways to Represent a Function

1. The functions $f(x) = x + \sqrt{2-x}$ and $g(u) = u + \sqrt{2-u}$ give exactly the same output values for every input value, so f and g are equal.
3. (a) The point $(1, 3)$ is on the graph of f , so $f(1) = 3$.
(b) When $x = -1$, y is about -0.2 , so $f(-1) \approx -0.2$.
(c) $f(x) = 1$ is equivalent to $y = 1$. When $y = 1$, we have $x = 0$ and $x = 3$.
(d) A reasonable estimate for x when $y = 0$ is $x = -0.8$.
(e) The domain of f consists of all x -values on the graph of f . For this function, the domain is $-2 \leq x \leq 4$, or $[-2, 4]$.
The range of f consists of all y -values on the graph of f . For this function, the range is $-1 \leq y \leq 3$, or $[-1, 3]$.
(f) As x increases from -2 to 1 , y increases from -1 to 3 . Thus, f is increasing on the interval $[-2, 1]$.
5. From Figure 1 in the text, the lowest point occurs at about $(t, a) = (12, -85)$. The highest point occurs at about $(17, 115)$. Thus, the range of the vertical ground acceleration is $-85 \leq a \leq 115$. Written in interval notation, we get $[-85, 115]$.
7. No, the curve is not the graph of a function because a vertical line intersects the curve more than once. Hence, the curve fails the Vertical Line Test.
9. Yes, the curve is the graph of a function because it passes the Vertical Line Test. The domain is $[-3, 2]$ and the range is $[-3, -2) \cup [-1, 3]$.
11. The person's weight increased to about 64 kg at age 20 and stayed fairly steady for 10 years. The person's weight dropped to about 52 kg for the next 5 years, then increased rapidly to about 68 kg. The next 30 years saw a gradual increase to 76 kg. Possible reasons for the drop in weight at 30 years of age: diet, exercise, health problems.
13. The water will cool down almost to freezing as the ice melts. Then, when the ice has melted, the water will slowly warm up to room temperature.
- 
15. (a) The power consumption at 6 AM is 500 MW, which is obtained by reading the value of power P when $t = 6$ from the graph. At 6 PM we read the value of P when $t = 18$, obtaining approximately 730 MW.
(b) The minimum power consumption is determined by finding the time for the lowest point on the graph, $t = 4$, or 4 AM. The maximum power consumption corresponds to the highest point on the graph, which occurs just before $t = 12$, or right before noon. These times are reasonable, considering the power consumption schedules of most individuals and businesses.

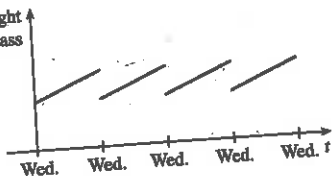
17. Of course, this graph depends strongly on the geographical location!



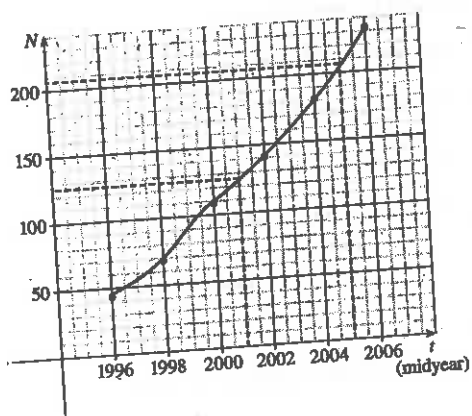
19. As the price increases, the amount sold decreases.



21. Height of grass



23. (a)



(b) From the graph, we estimate the number of US cell-phone subscribers to be about 126 million in 2001 and 207 million in 2005.

$$25. f(x) = 3x^2 - x + 2.$$

$$f(2) = 3(2)^2 - 2 + 2 = 12 - 2 + 2 = 12.$$

$$f(-2) = 3(-2)^2 - (-2) + 2 = 12 + 2 + 2 = 16.$$

$$f(a) = 3a^2 - a + 2.$$

$$f(-a) = 3(-a)^2 - (-a) + 2 = 3a^2 + a + 2.$$

$$f(a+1) = 3(a+1)^2 - (a+1) + 2 = 3(a^2 + 2a + 1) - a - 1 + 2 = 3a^2 + 6a + 3 - a + 1 = 3a^2 + 5a + 4.$$

$$2f(a) = 2 \cdot f(a) = 2(3a^2 - a + 2) = 6a^2 - 2a + 4.$$

$$f(2a) = 3(2a)^2 - (2a) + 2 = 3(4a^2) - 2a + 2 = 12a^2 - 2a + 2.$$

$$f(a^2) = 3(a^2)^2 - (a^2) + 2 = 3(a^4) - a^2 + 2 = 3a^4 - a^2 + 2.$$

$$[f(a)]^2 = [3a^2 - a + 2]^2 = (3a^2 - a + 2)(3a^2 - a + 2)$$

$$= 9a^4 - 3a^3 + 6a^2 - 3a^3 + a^2 - 2a + 6a^2 - 2a + 4 = 9a^4 - 6a^3 + 13a^2 - 4a + 4.$$

$$f(a+h) = 3(a+h)^2 - (a+h) + 2 = 3(a^2 + 2ah + h^2) - a - h + 2 = 3a^2 + 6ah + 3h^2 - a - h + 2.$$

27. $f(x) = 4 + 3x - x^2$, so $f(3+h) = 4 + 3(3+h) - (3+h)^2 = 4 + 9 + 3h - (9 + 6h + h^2) = 4 - 3h - h^2$,

and $\frac{f(3+h) - f(3)}{h} = \frac{(4 - 3h - h^2) - 4}{h} = \frac{h(-3-h)}{h} = -3 - h$.

29. $\frac{f(x) - f(a)}{x - a} = \frac{\frac{1}{x} - \frac{1}{a}}{x - a} = \frac{\frac{a-x}{xa}}{x-a} = \frac{a-x}{xa(x-a)} = \frac{-1(x-a)}{xa(x-a)} = -\frac{1}{ax}$

31. $f(x) = (x+4)/(x^2-9)$ is defined for all x except when $0 = x^2 - 9 \Leftrightarrow 0 = (x+3)(x-3) \Leftrightarrow x = -3$ or 3 , so the domain is $\{x \in \mathbb{R} \mid x \neq -3, 3\} = (-\infty, -3) \cup (-3, 3) \cup (3, \infty)$.

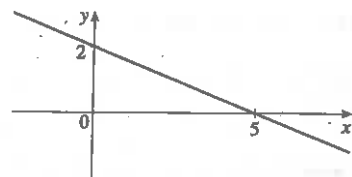
33. $f(t) = \sqrt[3]{2t-1}$ is defined for all real numbers. In fact $\sqrt[p]{p(t)}$, where $p(t)$ is a polynomial, is defined for all real numbers.

Thus, the domain is \mathbb{R} , or $(-\infty, \infty)$.

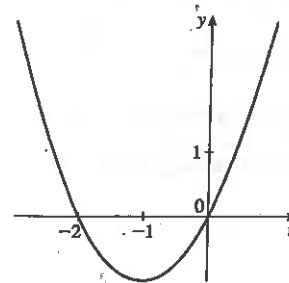
35. $h(x) = 1/\sqrt{x^2-5x}$ is defined when $x^2 - 5x > 0 \Leftrightarrow x(x-5) > 0$. Note that $x^2 - 5x \neq 0$ since that would result in division by zero. The expression $x(x-5)$ is positive if $x < 0$ or $x > 5$. (See Appendix A for methods for solving inequalities.) Thus, the domain is $(-\infty, 0) \cup (5, \infty)$.

37. $F(p) = \sqrt{2-\sqrt{p}}$ is defined when $p \geq 0$ and $2 - \sqrt{p} \geq 0$. Since $2 - \sqrt{p} \geq 0 \Rightarrow 2 \geq \sqrt{p} \Rightarrow \sqrt{p} \leq 2 \Rightarrow 0 \leq p \leq 4$, the domain is $[0, 4]$.

39. $f(x) = 2 - 0.4x$ is defined for all real numbers, so the domain is \mathbb{R} , or $(-\infty, \infty)$. The graph of f is a line with slope -0.4 and y -intercept 2 .

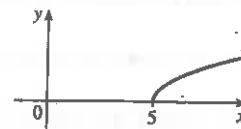


41. $f(t) = 2t + t^2$ is defined for all real numbers, so the domain is \mathbb{R} , or $(-\infty, \infty)$. The graph of f is a parabola opening upward since the coefficient of t^2 is positive. To find the t -intercepts, let $y = 0$ and solve for t . $0 = 2t + t^2 = t(2+t) \Rightarrow t = 0$ or $t = -2$. The t -coordinate of the vertex is halfway between the t -intercepts, that is, at $t = -1$. Since $f(-1) = 2(-1) + (-1)^2 = -2 + 1 = -1$, the vertex is $(-1, -1)$.



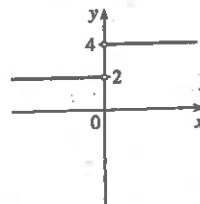
43. $g(x) = \sqrt{x-5}$ is defined when $x - 5 \geq 0$ or $x \geq 5$, so the domain is $[5, \infty)$.

Since $y = \sqrt{x-5} \Rightarrow y^2 = x-5 \Rightarrow x = y^2 + 5$, we see that g is the top half of a parabola.



45. $G(x) = \frac{3x + |x|}{x}$. Since $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$, we have

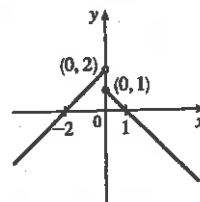
$$G(x) = \begin{cases} \frac{3x+x}{x} & \text{if } x > 0 \\ \frac{3x-x}{x} & \text{if } x < 0 \end{cases} = \begin{cases} \frac{4x}{x} & \text{if } x > 0 \\ \frac{2x}{x} & \text{if } x < 0 \end{cases} = \begin{cases} 4 & \text{if } x > 0 \\ 2 & \text{if } x < 0 \end{cases}$$



Note that G is not defined for $x = 0$. The domain is $(-\infty, 0) \cup (0, \infty)$.

47. $f(x) = \begin{cases} x+2 & \text{if } x < 0 \\ 1-x & \text{if } x \geq 0 \end{cases}$

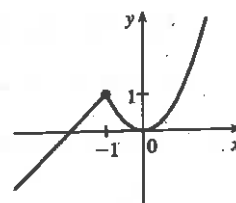
The domain is \mathbb{R} .



49. $f(x) = \begin{cases} x+2 & \text{if } x \leq -1 \\ x^2 & \text{if } x > -1 \end{cases}$

Note that for $x = -1$, both $x+2$ and x^2 are equal to 1.

The domain is \mathbb{R} .



51. Recall that the slope m of a line between the two points (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$ and an equation of the line

connecting those two points is $y - y_1 = m(x - x_1)$. The slope of the line segment joining the points $(1, -3)$ and $(5, 7)$ is

$$\frac{7 - (-3)}{5 - 1} = \frac{5}{2}, \text{ so an equation is } y - (-3) = \frac{5}{2}(x - 1). \text{ The function is } f(x) = \frac{5}{2}x - \frac{11}{2}, 1 \leq x \leq 5.$$

53. We need to solve the given equation for y . $x + (y - 1)^2 = 0 \Leftrightarrow (y - 1)^2 = -x \Leftrightarrow y - 1 = \pm\sqrt{-x} \Leftrightarrow$

$y = 1 \pm \sqrt{-x}$. The expression with the positive radical represents the top half of the parabola, and the one with the negative

radical represents the bottom half. Hence, we want $f(x) = 1 - \sqrt{-x}$. Note that the domain is $x \leq 0$.

55. For $0 \leq x \leq 3$, the graph is the line with slope -1 and y -intercept 3 , that is, $y = -x + 3$. For $3 < x \leq 5$, the graph is the line

with slope 2 passing through $(3, 0)$; that is, $y - 0 = 2(x - 3)$, or $y = 2x - 6$. So the function is

$$f(x) = \begin{cases} -x + 3 & \text{if } 0 \leq x \leq 3 \\ 2x - 6 & \text{if } 3 < x \leq 5 \end{cases}$$

57. Let the length and width of the rectangle be L and W . Then the perimeter is $2L + 2W = 20$ and the area is $A = LW$.

Solving the first equation for W in terms of L gives $W = \frac{20 - 2L}{2} = 10 - L$. Thus, $A(L) = L(10 - L) = 10L - L^2$. Since

lengths are positive, the domain of A is $0 < L < 10$. If we further restrict L to be larger than W , then $5 < L < 10$ would be the domain.

59. Let the length of a side of the equilateral triangle be x . Then by the Pythagorean Theorem, the height y of the triangle satisfies $y^2 + (\frac{1}{2}x)^2 = x^2$, so that $y^2 = x^2 - \frac{1}{4}x^2 = \frac{3}{4}x^2$ and $y = \frac{\sqrt{3}}{2}x$. Using the formula for the area A of a triangle,

$$A = \frac{1}{2}(\text{base})(\text{height}), \text{ we obtain } A(x) = \frac{1}{2}(x)\left(\frac{\sqrt{3}}{2}x\right) = \frac{\sqrt{3}}{4}x^2, \text{ with domain } x > 0.$$

61. Let each side of the base of the box have length x , and let the height of the box be h . Since the volume is 2, we know that $2 = hx^2$, so that $h = 2/x^2$, and the surface area is $S = x^2 + 4xh$. Thus, $S(x) = x^2 + 4x(2/x^2) = x^2 + (8/x)$, with domain $x > 0$.

63. The height of the box is x and the length and width are $L = 20 - 2x$, $W = 12 - 2x$. Then $V = LWx$ and so

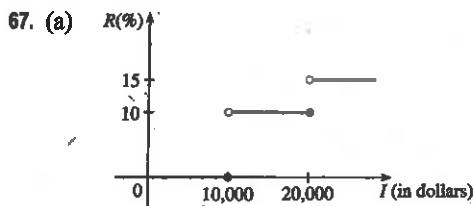
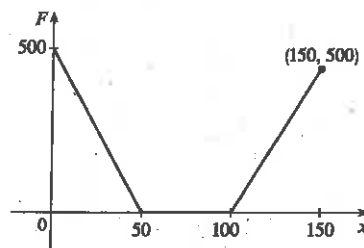
$$V(x) = (20 - 2x)(12 - 2x)(x) = 4(10 - x)(6 - x)(x) = 4x(60 - 16x + x^2) = 4x^3 - 64x^2 + 240x.$$

The sides L , W , and x must be positive. Thus, $L > 0 \Leftrightarrow 20 - 2x > 0 \Leftrightarrow x < 10$;

$W > 0 \Leftrightarrow 12 - 2x > 0 \Leftrightarrow x < 6$; and $x > 0$. Combining these restrictions gives us the domain $0 < x < 6$.

65. We can summarize the amount of the fine with a piecewise defined function.

$$F(x) = \begin{cases} 10(50 - x) & \text{if } 0 \leq x < 50 \\ 0 & \text{if } 50 \leq x \leq 100 \\ 10(x - 100) & \text{if } x > 100 \end{cases}$$

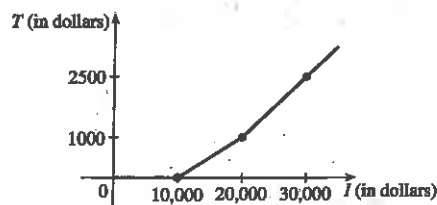


(b) On \$14,000, tax is assessed on \$4000, and $10\%(\$4000) = \400 .

On \$26,000, tax is assessed on \$16,000, and

$$10\%(\$10,000) + 15\%(\$6000) = \$1000 + \$900 = \$1900.$$

(c) As in part (b), there is \$1000 tax assessed on \$20,000 of income, so the graph of T is a line segment from $(10,000, 0)$ to $(20,000, 1000)$. The tax on \$30,000 is \$2500, so the graph of T for $x > 20,000$ is the ray with initial point $(20,000, 1000)$ that passes through $(30,000, 2500)$.



69. f is an odd function because its graph is symmetric about the origin. g is an even function because its graph is symmetric with respect to the y -axis.

71. (a) Because an even function is symmetric with respect to the y -axis, and the point $(5, 3)$ is on the graph of this even function, the point $(-5, 3)$ must also be on its graph.

(b) Because an odd function is symmetric with respect to the origin, and the point $(5, 3)$ is on the graph of this odd function, the point $(-5, -3)$ must also be on its graph.