**Quiz #3** Name: Corrigé

Tangentes et Dérivées

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1. **Using the definition of the slope of a tangent**, find the **equation** of the tangent line to $\overset{f(x)}{y} = 7 + 2x - 3x^2$ at the point (-1, 2).

$$m = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{7 + 2(x+h) - 3(x+h)^2 - (7 + 2x - 3x^2)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{7} + \cancel{2x} + 2h - 3(x^2 + 2xh + h^2) - \cancel{7} - \cancel{2x} + 3x^2}{h}$$

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$$= \lim_{h \to 0} \frac{2h - \cancel{3x^2} - 6xh - 3h^2 + \cancel{3x^2}}{h}$$

At $(-1,2)$: $m = 2 - 6(-1)$

$m = 8$

$$= \lim_{h \to 0} \frac{\cancel{h}(2 - 6x - 3h)}{\cancel{h}}$$

$$\boxed{y - 2 = 8(x+1)}$$

$$= 2 - 6x - 3(0)$$

$$= 2 - 6x$$

2. Use the **definition of the derivative** to find the derivative of $f(x) = \sqrt{x-4}$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \left[ \frac{\sqrt{x+h-4} - \sqrt{x-4}}{h} \cdot \frac{\sqrt{x+h-4} + \sqrt{x-4}}{\sqrt{x+h-4} + \sqrt{x-4}} \right]$$

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$$= \lim_{h \to 0} \frac{\cancel{x} + \cancel{h} - \cancel{4} - (\cancel{x} - \cancel{4})}{\cancel{h}(\sqrt{x+h-4} + \sqrt{x-4})} = \frac{1}{\sqrt{x+0-4} + \sqrt{x-4}} = \boxed{\frac{1}{2\sqrt{x-4}}}$$

3. Find the points on the curve $y = \dfrac{\overset{f(x)}{x^2}}{\underset{g(x)}{x-3}}$ where the tangent is horizontal.

$m = 0$

$$y' = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

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$$y' = \frac{2x(x-3) - x^2(1)}{(x-3)^2}$$

$$y' = \frac{2x^2 - 6x - x^2}{(x-3)^2} = \frac{x^2 - 6x}{(x-3)^2}$$

$$(x-3)^2 \cdot 0 = \frac{x^2 - 6x}{\cancel{(x-3)^2}} \cdot \cancel{(x-3)^2}$$

$$0 = x(x-6)$$

$x = 0$ $\quad$ $x = 6$

$y = 0$ $\quad$ $y = \dfrac{6^2}{6-3} = \dfrac{36}{3} = 12$

$\boxed{(0,0)}$ $\quad$ $\boxed{(6,12)}$

4. Find the derivatives of the following functions.

a) $y = 5 - 3\sqrt{x} + 5x^3 - \frac{2}{x^2} \rightarrow 2x^{-2}$

/4 $y' = 0 - 3\left(\frac{1}{2}x^{-1/2}\right) + 5(3x^2) - 2(-2x^{-3})$

b) $g(x) = \frac{\overset{f(x)}{(6-2x+x^2)}}{\underset{h(x)}{(3-x-x^2)(4-3x^4)}} \rightarrow h(x) = a(x)b(x)$

/4 $g'(x) = \frac{f'(x)h(x) - f(x)h'(x)}{(h(x))^2}$

$g'(x) = \frac{f'(x)h(x) - f(x)[a'(x)b(x) + a(x)b'(x)]}{(h(x))^2}$

$g'(x) = \frac{(0-2+2x)(3-x-x^2)(4-3x^4) - (6-2x+x^2)[(0-1-2x)(4-3x^4) + (3-x-x^2)(0-3(4x^3))]}{[(3-x-x^2)(4-3x^4)]^2}$

5. If $f$ and $g$ are differentiable functions, prove that

$$(f(x)+g(x))' = f'(x)+g'(x)$$

/3 $(f(x)+g(x))' = \lim_{h\to 0} \frac{f(x+h)+g(x+h) - (f(x)+g(x))}{h}$

$= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} + \lim_{h\to 0} \frac{g(x+h)-g(x)}{h}$

$= f'(x) + g'(x)$