3.5 Exercises

1–4
(a) Find y' by implicit differentiation.
(b) Solve the equation explicitly for y and differentiate to get y' in terms of x.
(c) Check that your solutions to parts (a) and (b) are consistent by substituting the expression for y into your solution for part (a).

1. $xy + 2x + 3x^2 = 4$
2. $4x^2 + 9y^2 = 36$
3. $\frac{1}{x} + \frac{1}{y} = 1$
4. $\cos x + \sqrt{y} = 5$

5–20 Find dy/dx by implicit differentiation.

5. $x^3 + y^3 = 1$
6. $2\sqrt{x} + \sqrt{y} = 3$
7. $x^2 + xy - y^2 = 4$
8. $2x^3 + x^2y - xy^3 = 2$
9. $x^4(x + y) = y^2(3x - y)$
10. $xe^y = x - y$
11. $x^2y^2 + x \sin y = 4$
12. $1 + x = \sin(xy^2)$
13. $4 \cos x \sin y = 1$
14. $e^y \sin x = x + xy$
15. $e^{x/y} = x - y$
16. $\sqrt{x + y} = 1 + x^2y^2$
17. $\tan^{-1}(x^2y) = x + xy^2$
18. $x \sin y + y \sin x = 1$
19. $e^y \cos x = 1 + \sin(xy)$
20. $\tan(x - y) = \frac{y}{1 + x^2}$

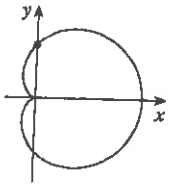
21. If $f(x) + x^2[f(x)]^3 = 10$ and $f(1) = 2$, find $f'(1)$.
22. If $g(x) + x \sin g(x) = x^2$, find $g'(0)$.

23–24 Regard y as the independent variable and x as the dependent variable and use implicit differentiation to find dx/dy.

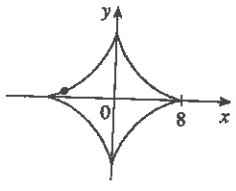
23. $x^4y^2 - x^3y + 2xy^3 = 0$
24. $y \sec x = x \tan y$

25–32 Use implicit differentiation to find an equation of the tangent line to the curve at the given point.

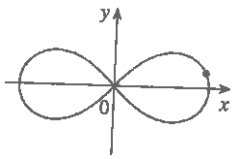
25. $y \sin 2x = x \cos 2y$, $(\pi/2, \pi/4)$
26. $\sin(x + y) = 2x - 2y$, (π, π)
27. $x^2 + xy + y^2 = 3$, $(1, 1)$ (ellipse)
28. $x^2 + 2xy - y^2 + x = 2$, $(1, 2)$ (hyperbola)
29. $x^2 + y^2 = (2x^2 + 2y^2 - x)^2$, $(0, \frac{1}{2})$ (cardioid)



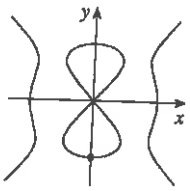
30. $x^{2/3} + y^{2/3} = 4$, $(-3\sqrt{3}, 1)$ (astroid)



31. $2(x^2 + y^2)^2 = 25(x^2 - y^2)$, $(3, 1)$ (lemniscate)



32. $y^2(y^2 - 4) = x^2(x^2 - 5)$, $(0, -2)$ (devil's curve)



33. (a) The curve with equation $y^2 = 5x^4 - x^2$ is called a **kampyle of Eudoxus**. Find an equation of the tangent line to this curve at the point $(1, 2)$.
(b) Illustrate part (a) by graphing the curve and the tangent line on a common screen. (If your graphing device will graph implicitly defined curves, then use that capability. If not, you can still graph this curve by graphing its upper and lower halves separately.)

34. (a) The curve with equation $y^2 = x^3 + 3x^2$ is called the **Tschirnhausen cubic**. Find an equation of the tangent line to this curve at the point $(1, -2)$.
(b) At what points does this curve have horizontal tangents?
(c) Illustrate parts (a) and (b) by graphing the curve and the tangent lines on a common screen.

35–38 Find y'' by implicit differentiation.

35. $9x^2 + y^2 = 9$
36. $\sqrt{x} + \sqrt{y} = 1$
37. $x^3 + y^3 = 1$
38. $x^4 + y^4 = a^4$

39. If $xy + e^y = e$, find the value of y'' at the point where $x = 0$.
40. If $x^2 + xy + y^3 = 1$, find the value of y''' at the point where $x = 1$.

CAS 41. Fanciful shapes can be created by using the implicit plotting capabilities of computer algebra systems.
(a) Graph the curve with equation

$$y(y^2 - 1)(y - 2) = x(x - 1)(x - 2)$$

At how many points does this curve have horizontal tangents? Estimate the x-coordinates of these points.
(b) Find equations of the tangent lines at the points $(0, 1)$ and $(0, 2)$.
(c) Find the exact x-coordinates of the points in part (a).
(d) Create even more fanciful curves by modifying the equation in part (a).

Graphing calculator or computer required CAS Computer algebra system required 1. Homework Hints available at stewartcalculus.com