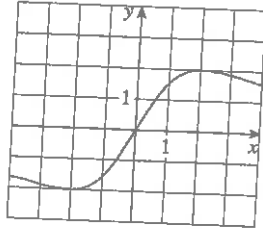


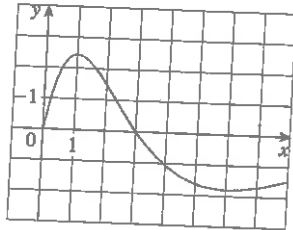
2.8 Exercises

1–2 Use the given graph to estimate the value of each derivative. Then sketch the graph of  $f'$ .

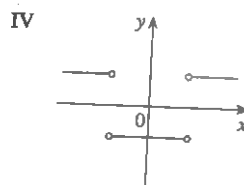
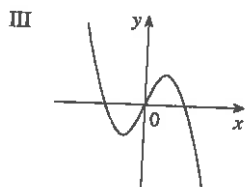
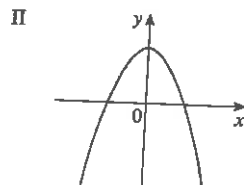
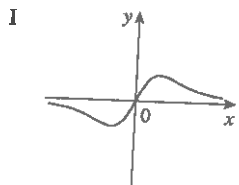
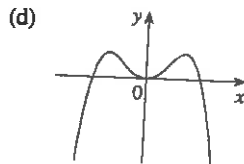
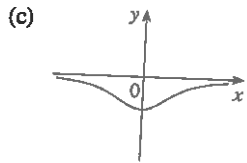
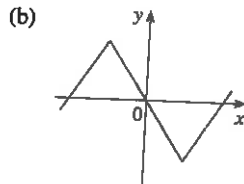
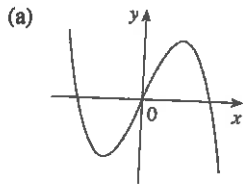
1. (a)  $f'(-3)$
- (b)  $f'(-2)$
- (c)  $f'(-1)$
- (d)  $f'(0)$
- (e)  $f'(1)$
- (f)  $f'(2)$
- (g)  $f'(3)$



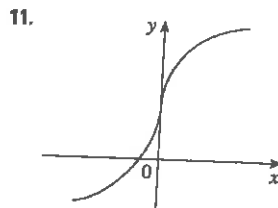
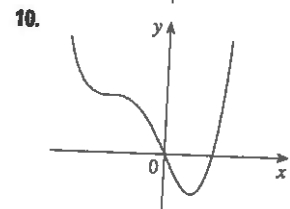
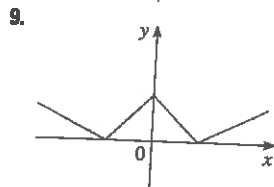
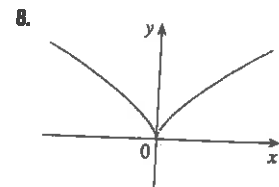
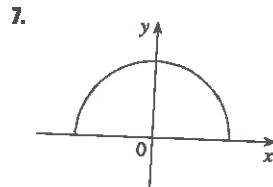
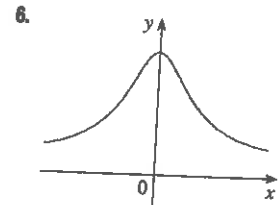
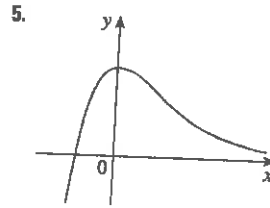
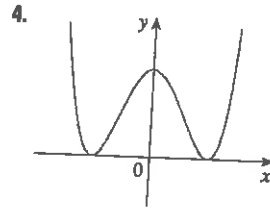
2. (a)  $f'(0)$
- (b)  $f'(1)$
- (c)  $f'(2)$
- (d)  $f'(3)$
- (e)  $f'(4)$
- (f)  $f'(5)$
- (g)  $f'(6)$
- (h)  $f'(7)$



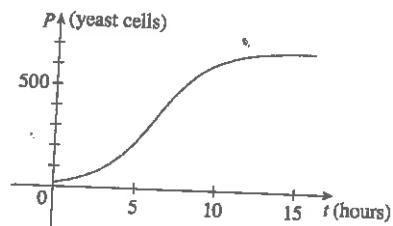
3. Match the graph of each function in (a)–(d) with the graph of its derivative in I–IV. Give reasons for your choices.



4–11 Trace or copy the graph of the given function  $f$ . (Assume that the axes have equal scales.) Then use the method of Example 1 to sketch the graph of  $f'$  below it.



12. Shown is the graph of the population function  $P(t)$  for yeast cells in a laboratory culture. Use the method of Example 1 to

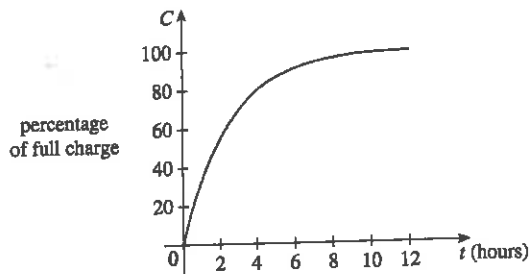


Graphing calculator or computer required

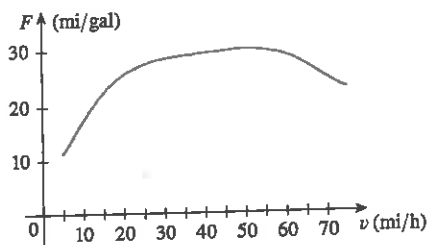
1. Homework Hints available at [stewartcalculus.com](http://stewartcalculus.com)

graph the derivative  $P'(t)$ . What does the graph of  $P'$  tell us about the yeast population?

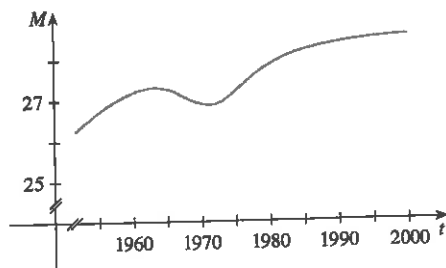
13. A rechargeable battery is plugged into a charger. The graph shows  $C(t)$ , the percentage of full capacity that the battery reaches as a function of time  $t$  elapsed (in hours).
- What is the meaning of the derivative  $C'(t)$ ?
  - Sketch the graph of  $C'(t)$ . What does the graph tell you?



14. The graph (from the US Department of Energy) shows how driving speed affects gas mileage. Fuel economy  $F$  is measured in miles per gallon and speed  $v$  is measured in miles per hour.
- What is the meaning of the derivative  $F'(v)$ ?
  - Sketch the graph of  $F'(v)$ .
  - At what speed should you drive if you want to save on gas?



15. The graph shows how the average age of first marriage of Japanese men varied in the last half of the 20th century. Sketch the graph of the derivative function  $M'(t)$ . During which years was the derivative negative?



16–18 Make a careful sketch of the graph of  $f$  and below it sketch the graph of  $f'$  in the same manner as in Exercises 4–11. Can you guess a formula for  $f'(x)$  from its graph?

16.  $f(x) = \sin x$

17.  $f(x) = e^x$

18.  $f(x) = \ln x$

19. Let  $f(x) = x^2$ .
- Estimate the values of  $f'(0)$ ,  $f'(\frac{1}{2})$ ,  $f'(1)$ , and  $f'(2)$  by using a graphing device to zoom in on the graph of  $f$ .
  - Use symmetry to deduce the values of  $f'(-\frac{1}{2})$ ,  $f'(-1)$ , and  $f'(-2)$ .
  - Use the results from parts (a) and (b) to guess a formula for  $f'(x)$ .
  - Use the definition of derivative to prove that your guess in part (c) is correct.

20. Let  $f(x) = x^3$ .
- Estimate the values of  $f'(0)$ ,  $f'(\frac{1}{2})$ ,  $f'(1)$ ,  $f'(2)$ , and  $f'(3)$  by using a graphing device to zoom in on the graph of  $f$ .
  - Use symmetry to deduce the values of  $f'(-\frac{1}{2})$ ,  $f'(-1)$ ,  $f'(-2)$ , and  $f'(-3)$ .
  - Use the values from parts (a) and (b) to graph  $f'$ .
  - Guess a formula for  $f'(x)$ .
  - Use the definition of derivative to prove that your guess in part (d) is correct.

21–31 Find the derivative of the function using the definition of derivative. State the domain of the function and the domain of its derivative.

21.  $f(x) = \frac{1}{2}x - \frac{1}{3}$

22.  $f(x) = mx + b$

23.  $f(t) = 5t - 9t^2$

24.  $f(x) = 1.5x^2 - x + 3.7$

25.  $f(x) = x^3 - 3x + 5$

26.  $f(x) = x + \sqrt{x}$

27.  $g(x) = \sqrt{9 - x}$

28.  $f(x) = \frac{x^2 - 1}{2x - 3}$

29.  $G(t) = \frac{1 - 2t}{3 + t}$

30.  $f(x) = x^{3/2}$

31.  $f(x) = x^4$

32. (a) Sketch the graph of  $f(x) = \sqrt{6 - x}$  by starting with the graph of  $y = \sqrt{x}$  and using the transformations of Section 1.3.  
 (b) Use the graph from part (a) to sketch the graph of  $f'$ .  
 (c) Use the definition of a derivative to find  $f'(x)$ . What are the domains of  $f$  and  $f'$ ?
- (d) Use a graphing device to graph  $f'$  and compare with your sketch in part (b).

33. (a) If  $f(x) = x^4 + 2x$ , find  $f'(x)$ .  
 (b) Check to see that your answer to part (a) is reasonable by comparing the graphs of  $f$  and  $f'$ .

34. (a) If  $f(x) = x + 1/x$ , find  $f'(x)$ .  
 (b) Check to see that your answer to part (a) is reasonable by comparing the graphs of  $f$  and  $f'$ .

35. The unemployment rate  $U(t)$  varies with time. The table gives the percentage of unemployed in the Australian labor force measured at midyear from 1995 to 2004.

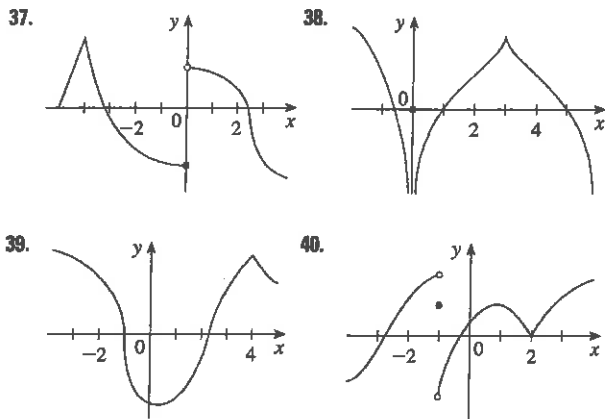
$t$	$U(t)$	$t$	$U(t)$
1995	8.1	2000	6.2
1996	8.0	2001	6.9
1997	8.2	2002	6.5
1998	7.9	2003	6.2
1999	6.7	2004	5.6

- (a) What is the meaning of  $U'(t)$ ? What are its units?  
 (b) Construct a table of estimated values for  $U'(t)$ .
36. Let  $P(t)$  be the percentage of the population of the Philippines over the age of 60 at time  $t$ . The table gives projections of values of this function from 1995 to 2020.

$t$	$P(t)$	$t$	$P(t)$
1995	5.2	2010	6.7
2000	5.5	2015	7.7
2005	6.1	2020	8.9

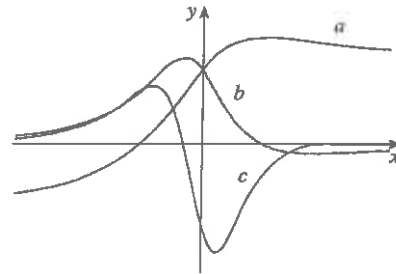
- (a) What is the meaning of  $P'(t)$ ? What are its units?  
 (b) Construct a table of estimated values for  $P'(t)$ .  
 (c) Graph  $P$  and  $P'$ .

37–40 The graph of  $f$  is given. State, with reasons, the numbers at which  $f$  is not differentiable.

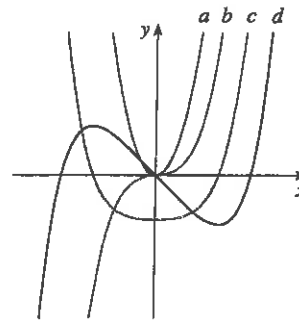


41. Graph the function  $f(x) = x + \sqrt{|x|}$ . Zoom in repeatedly, first toward the point  $(-1, 0)$  and then toward the origin. What is different about the behavior of  $f$  in the vicinity of these two points? What do you conclude about the differentiability of  $f$ ?
42. Zoom in toward the points  $(1, 0)$ ,  $(0, 1)$ , and  $(-1, 0)$  on the graph of the function  $g(x) = (x^2 - 1)^{2/3}$ . What do you notice? Account for what you see in terms of the differentiability of  $g$ .

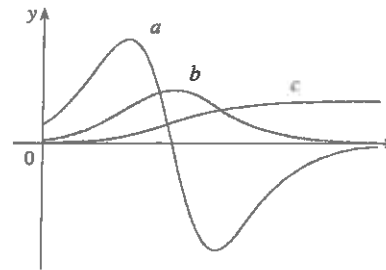
43. The figure shows the graphs of  $f$ ,  $f'$ , and  $f''$ . Identify each curve, and explain your choices.



44. The figure shows graphs of  $f$ ,  $f'$ ,  $f''$ , and  $f'''$ . Identify each curve, and explain your choices.



45. The figure shows the graphs of three functions. One is the position function of a car, one is the velocity of the car, and one is its acceleration. Identify each curve, and explain your choices.



46. The figure shows the graphs of four functions. One is the position function of a car, one is the velocity of the car, one is its acceleration, and one is its jerk. Identify each curve, and explain your choices.

