

2.2 Exercises

1. Explain in your own words what is meant by the equation

$$\lim_{x \rightarrow 2} f(x) = 5$$

Is it possible for this statement to be true and yet  $f(2) = 3$ ? Explain.

2. Explain what it means to say that

$$\lim_{x \rightarrow 1^-} f(x) = 3 \quad \text{and} \quad \lim_{x \rightarrow 1^+} f(x) = 7$$

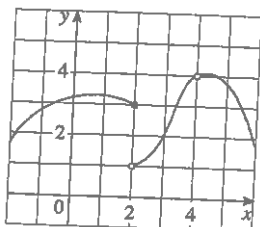
In this situation is it possible that  $\lim_{x \rightarrow 1} f(x)$  exists? Explain.

3. Explain the meaning of each of the following.

(a)  $\lim_{x \rightarrow -3} f(x) = \infty$       (b)  $\lim_{x \rightarrow 4^+} f(x) = -\infty$

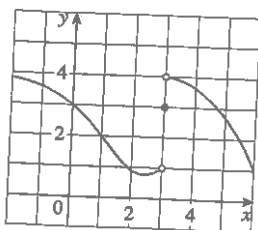
4. Use the given graph of  $f$  to state the value of each quantity, if it exists. If it does not exist, explain why.

(a)  $\lim_{x \rightarrow 2^-} f(x)$     (b)  $\lim_{x \rightarrow 2^+} f(x)$     (c)  $\lim_{x \rightarrow 2} f(x)$   
 (d)  $f(2)$       (e)  $\lim_{x \rightarrow 4} f(x)$     (f)  $f(4)$



5. For the function  $f$  whose graph is given, state the value of each quantity, if it exists. If it does not exist, explain why.

(a)  $\lim_{x \rightarrow 1} f(x)$     (b)  $\lim_{x \rightarrow 3^-} f(x)$     (c)  $\lim_{x \rightarrow 3^+} f(x)$   
 (d)  $\lim_{x \rightarrow 3} f(x)$     (e)  $f(3)$

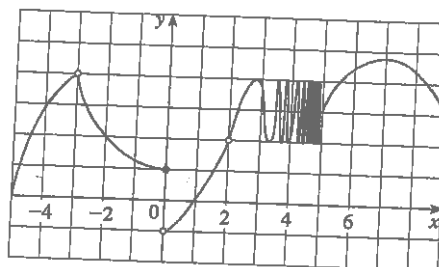


6. For the function  $h$  whose graph is given, state the value of each quantity, if it exists. If it does not exist, explain why.

(a)  $\lim_{x \rightarrow -3^-} h(x)$     (b)  $\lim_{x \rightarrow -3^+} h(x)$     (c)  $\lim_{x \rightarrow -3} h(x)$

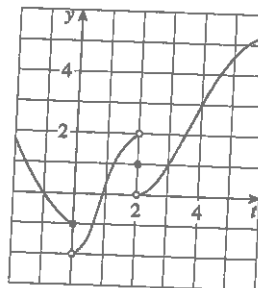
Graphing calculator or computer required

(d)  $h(-3)$       (e)  $\lim_{x \rightarrow 0^-} h(x)$     (f)  $\lim_{x \rightarrow 0^+} h(x)$   
 (g)  $\lim_{x \rightarrow 0} h(x)$     (h)  $h(0)$       (i)  $\lim_{x \rightarrow 2} h(x)$   
 (j)  $h(2)$       (k)  $\lim_{x \rightarrow 5^+} h(x)$     (l)  $\lim_{x \rightarrow 5^-} h(x)$



7. For the function  $g$  whose graph is given, state the value of each quantity, if it exists. If it does not exist, explain why.

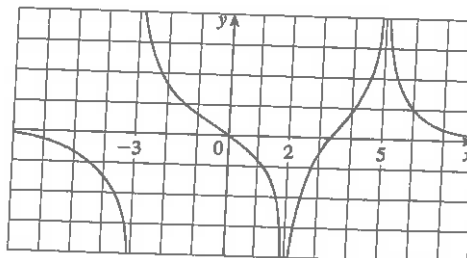
(a)  $\lim_{t \rightarrow 0^-} g(t)$     (b)  $\lim_{t \rightarrow 0^+} g(t)$     (c)  $\lim_{t \rightarrow 0} g(t)$   
 (d)  $\lim_{t \rightarrow 2^-} g(t)$     (e)  $\lim_{t \rightarrow 2^+} g(t)$     (f)  $\lim_{t \rightarrow 2} g(t)$   
 (g)  $g(2)$       (h)  $\lim_{t \rightarrow 4} g(t)$



8. For the function  $R$  whose graph is shown, state the following.

(a)  $\lim_{x \rightarrow 2} R(x)$       (b)  $\lim_{x \rightarrow 5} R(x)$   
 (c)  $\lim_{x \rightarrow -3^-} R(x)$       (d)  $\lim_{x \rightarrow -3^+} R(x)$

- (e) The equations of the vertical asymptotes.



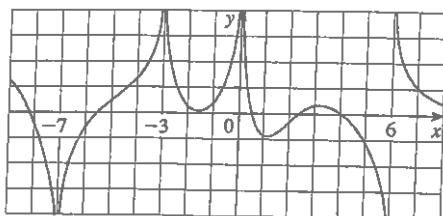
1. Homework Hints available at [stewartcalculus.com](http://stewartcalculus.com)

9. For the function  $f$  whose graph is shown, state the following.

(a)  $\lim_{x \rightarrow -7} f(x)$     (b)  $\lim_{x \rightarrow -3} f(x)$     (c)  $\lim_{x \rightarrow 0} f(x)$

(d)  $\lim_{x \rightarrow 6^-} f(x)$     (e)  $\lim_{x \rightarrow 6^+} f(x)$

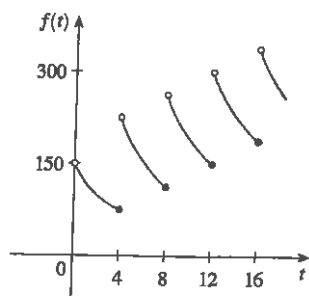
(f) The equations of the vertical asymptotes.



10. A patient receives a 150-mg injection of a drug every 4 hours. The graph shows the amount  $f(t)$  of the drug in the bloodstream after  $t$  hours. Find

$$\lim_{t \rightarrow 12^-} f(t) \quad \text{and} \quad \lim_{t \rightarrow 12^+} f(t)$$

and explain the significance of these one-sided limits.



11–12 Sketch the graph of the function and use it to determine the values of  $a$  for which  $\lim_{x \rightarrow a} f(x)$  exists.

$$11. f(x) = \begin{cases} 1 + x & \text{if } x < -1 \\ x^2 & \text{if } -1 \leq x < 1 \\ 2 - x & \text{if } x \geq 1 \end{cases}$$

$$12. f(x) = \begin{cases} 1 + \sin x & \text{if } x < 0 \\ \cos x & \text{if } 0 \leq x \leq \pi \\ \sin x & \text{if } x > \pi \end{cases}$$

13–14 Use the graph of the function  $f$  to state the value of each limit, if it exists. If it does not exist, explain why.

(a)  $\lim_{x \rightarrow 0^-} f(x)$     (b)  $\lim_{x \rightarrow 0^+} f(x)$     (c)  $\lim_{x \rightarrow 0} f(x)$

13.  $f(x) = \frac{1}{1 + e^{1/x}}$

14.  $f(x) = \frac{x^2 + x}{\sqrt{x^3 + x^2}}$

15–18 Sketch the graph of an example of a function  $f$  that satisfies all of the given conditions.

15.  $\lim_{x \rightarrow 1^-} f(x) = 2$ ,  $\lim_{x \rightarrow 1^+} f(x) = -2$ ,  $f(1) = 2$

16.  $\lim_{x \rightarrow 0^-} f(x) = 1$ ,  $\lim_{x \rightarrow 0^+} f(x) = -1$ ,  $\lim_{x \rightarrow 2} f(x) = 0$ ,  
 $\lim_{x \rightarrow 2^+} f(x) = 1$ ,  $f(2) = 1$ ,  $f(0)$  is undefined

17.  $\lim_{x \rightarrow 3^+} f(x) = 4$ ,  $\lim_{x \rightarrow 3^-} f(x) = 2$ ,  $\lim_{x \rightarrow -2} f(x) = 2$ ,  
 $f(3) = 3$ ,  $f(-2) = 1$

18.  $\lim_{x \rightarrow 0^-} f(x) = 2$ ,  $\lim_{x \rightarrow 0^+} f(x) = 0$ ,  $\lim_{x \rightarrow 4^-} f(x) = 3$ ,  
 $\lim_{x \rightarrow 4^+} f(x) = 0$ ,  $f(0) = 2$ ,  $f(4) = 1$

19–22 Guess the value of the limit (if it exists) by evaluating the function at the given numbers (correct to six decimal places).

19.  $\lim_{x \rightarrow 2} \frac{x^2 - 2x}{x^2 - x - 2}$ ,  
 $x = 2.5, 2.1, 2.05, 2.01, 2.005, 2.001,$   
 $1.9, 1.95, 1.99, 1.995, 1.999$

20.  $\lim_{x \rightarrow -1} \frac{x^2 - 2x}{x^2 - x - 2}$ ,  
 $x = 0, -0.5, -0.9, -0.95, -0.99, -0.999,$   
 $-2, -1.5, -1.1, -1.01, -1.001$

21.  $\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2}$ ,  $x = \pm 1, \pm 0.5, \pm 0.1, \pm 0.05, \pm 0.01$

22.  $\lim_{x \rightarrow 0^+} x \ln(x + x^2)$ ,  $x = 1, 0.5, 0.1, 0.05, 0.01, 0.005, 0.001$

23–26 Use a table of values to estimate the value of the limit. If you have a graphing device, use it to confirm your result graphically.

23.  $\lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{x}$

24.  $\lim_{x \rightarrow 0} \frac{\tan 3x}{\tan 5x}$

25.  $\lim_{x \rightarrow 1} \frac{x^6 - 1}{x^{10} - 1}$

26.  $\lim_{x \rightarrow 0} \frac{9^x - 5^x}{x}$

27. (a) By graphing the function  $f(x) = (\cos 2x - \cos x)/x^2$  and zooming in toward the point where the graph crosses the  $y$ -axis, estimate the value of  $\lim_{x \rightarrow 0} f(x)$ .  
(b) Check your answer in part (a) by evaluating  $f(x)$  for values of  $x$  that approach 0.

28. (a) Estimate the value of

$$\lim_{x \rightarrow 0} \frac{\sin x}{\sin \pi x}$$

by graphing the function  $f(x) = (\sin x)/(\sin \pi x)$ . State your answer correct to two decimal places.

- (b) Check your answer in part (a) by evaluating  $f(x)$  for values of  $x$  that approach 0.

29–37 Determine the infinite limit.

29.  $\lim_{x \rightarrow -3^+} \frac{x+2}{x+3}$

30.  $\lim_{x \rightarrow -3^-} \frac{x+2}{x+3}$

31.  $\lim_{x \rightarrow 1} \frac{2-x}{(x-1)^2}$

32.  $\lim_{x \rightarrow 5^-} \frac{e^x}{(x-5)^3}$

33.  $\lim_{x \rightarrow 3^+} \ln(x^2 - 9)$

34.  $\lim_{x \rightarrow \pi^-} \cot x$

35.  $\lim_{x \rightarrow 2\pi^-} x \csc x$

36.  $\lim_{x \rightarrow 2^-} \frac{x^2 - 2x}{x^2 - 4x + 4}$

37.  $\lim_{x \rightarrow 2^+} \frac{x^2 - 2x - 8}{x^2 - 5x + 6}$

38. (a) Find the vertical asymptotes of the function

$$y = \frac{x^2 + 1}{3x - 2x^2}$$

- (b) Confirm your answer to part (a) by graphing the function.

39. Determine  $\lim_{x \rightarrow 1^-} \frac{1}{x^3 - 1}$  and  $\lim_{x \rightarrow 1^+} \frac{1}{x^3 - 1}$

(a) by evaluating  $f(x) = 1/(x^3 - 1)$  for values of  $x$  that approach 1 from the left and from the right,

(b) by reasoning as in Example 9, and

(c) from a graph of  $f$ .

40. (a) By graphing the function  $f(x) = (\tan 4x)/x$  and zooming in toward the point where the graph crosses the  $y$ -axis, estimate the value of  $\lim_{x \rightarrow 0} f(x)$ .

(b) Check your answer in part (a) by evaluating  $f(x)$  for values of  $x$  that approach 0.

41. (a) Estimate the value of the limit  $\lim_{x \rightarrow 0} (1+x)^{1/x}$  to five decimal places. Does this number look familiar?

(b) Illustrate part (a) by graphing the function  $y = (1+x)^{1/x}$ .

42. (a) Graph the function  $f(x) = e^x + \ln|x-4|$  for  $0 \leq x \leq 5$ . Do you think the graph is an accurate representation of  $f$ ?

(b) How would you get a graph that represents  $f$  better?

43. (a) Evaluate the function  $f(x) = x^2 - (2^x/1000)$  for  $x = 1, 0.8, 0.6, 0.4, 0.2, 0.1,$  and  $0.05$ , and guess the value of

$$\lim_{x \rightarrow 0} \left( x^2 - \frac{2^x}{1000} \right)$$

- (b) Evaluate  $f(x)$  for  $x = 0.04, 0.02, 0.01, 0.005, 0.003,$  and  $0.001$ . Guess again.

44. (a) Evaluate  $h(x) = (\tan x - x)/x^3$  for  $x = 1, 0.5, 0.1, 0.05, 0.01,$  and  $0.005$ .

(b) Guess the value of  $\lim_{x \rightarrow 0} \frac{\tan x - x}{x^3}$ .

(c) Evaluate  $h(x)$  for successively smaller values of  $x$  until you finally reach a value of 0 for  $h(x)$ . Are you still confident that your guess in part (b) is correct? Explain why you eventually obtained 0 values. (In Section 4.4 a method for evaluating the limit will be explained.)

- (d) Graph the function  $h$  in the viewing rectangle  $[-1, 1]$  by  $[0, 1]$ . Then zoom in toward the point where the graph crosses the  $y$ -axis to estimate the limit of  $h(x)$  as  $x$  approaches 0. Continue to zoom in until you observe distortions in the graph of  $h$ . Compare with the results of part (c).

45. Graph the function  $f(x) = \sin(\pi/x)$  of Example 4 in the viewing rectangle  $[-1, 1]$  by  $[-1, 1]$ . Then zoom in toward the origin several times. Comment on the behavior of this function.

46. In the theory of relativity, the mass of a particle with velocity  $v$  is

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

where  $m_0$  is the mass of the particle at rest and  $c$  is the speed of light. What happens as  $v \rightarrow c^-$ ?

47. Use a graph to estimate the equations of all the vertical asymptotes of the curve

$$y = \tan(2 \sin x) \quad -\pi \leq x \leq \pi$$

Then find the exact equations of these asymptotes.

48. (a) Use numerical and graphical evidence to guess the value of the limit

$$\lim_{x \rightarrow 1} \frac{x^3 - 1}{\sqrt{x} - 1}$$

- (b) How close to 1 does  $x$  have to be to ensure that the function in part (a) is within a distance 0.5 of its limit?