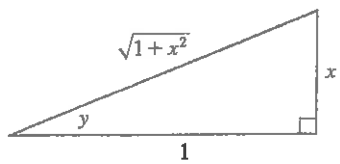


FIGURE 24

SOLUTION 2 Instead of using trigonometric identities as in Solution 1, it is perhaps easier to use a diagram. If $y = \tan^{-1}x$, then $\tan y = x$, and we can read from Figure 24 (which illustrates the case $y > 0$) that

$$\cos(\tan^{-1}x) = \cos y = \frac{1}{\sqrt{1 + x^2}}$$

The inverse tangent function, $\tan^{-1} = \arctan$, has domain $\mathbb{R}$ and range $(-\pi/2, \pi/2)$. Its graph is shown in Figure 25.

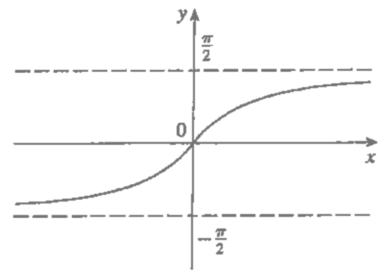


FIGURE 25
$y = \tan^{-1} x = \arctan x$

We know that the lines $x = \pm\pi/2$ are vertical asymptotes of the graph of tan. Since the graph of $\tan^{-1}$ is obtained by reflecting the graph of the restricted tangent function about the line $y = x$, it follows that the lines $y = \pi/2$ and $y = -\pi/2$ are horizontal asymptotes of the graph of $\tan^{-1}$.

The remaining inverse trigonometric functions are not used as frequently and are summarized here.

11
$$y = \csc^{-1}x \ (|x| \geq 1) \iff \csc y = x \text{ and } y \in (0, \pi/2] \cup (\pi, 3\pi/2]$$
$$y = \sec^{-1}x \ (|x| \geq 1) \iff \sec y = x \text{ and } y \in [0, \pi/2) \cup [\pi, 3\pi/2)$$
$$y = \cot^{-1}x \ (x \in \mathbb{R}) \iff \cot y = x \text{ and } y \in (0, \pi)$$

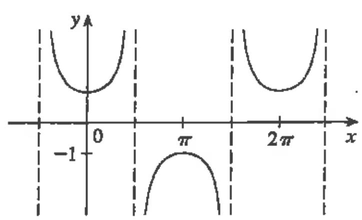


FIGURE 26
$y = \sec x$

The choice of intervals for y in the definitions of $\csc^{-1}$ and $\sec^{-1}$ is not universally agreed upon. For instance, some authors use $y \in [0, \pi/2) \cup (\pi/2, \pi]$ in the definition of $\sec^{-1}$. (You can see from the graph of the secant function in Figure 26 that both this choice and the one in **11** will work.)

1.6 Exercises

1. (a) What is a one-to-one function?
   (b) How can you tell from the graph of a function whether it is one-to-one?

2. (a) Suppose f is a one-to-one function with domain A and range B. How is the inverse function f^{-1} defined? What is the domain of f^{-1}? What is the range of f^{-1}?
   (b) If you are given a formula for f, how do you find a formula for f^{-1}?
   (c) If you are given the graph of f, how do you find the graph of f^{-1}?

3–14 A function is given by a table of values, a graph, a formula, or a verbal description. Determine whether it is one-to-one.

3.

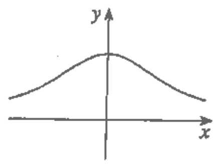
x	1	2	3	4	5	6
$f(x)$	1.5	2.0	3.6	5.3	2.8	2.0

4.

x	1	2	3	4	5	6
$f(x)$	1.0	1.9	2.8	3.5	3.1	2.9

Graphing calculator or computer required   CAS Computer algebra system required   1. Homework Hints available at stewartcalculus.com

5.



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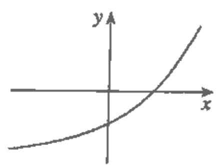

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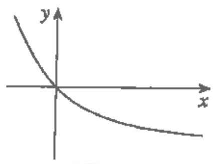

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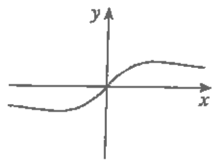


9. $f(x) = \frac{1}{2}(x + 5)$

10. $f(x) = 1 + 4x - x^2$

11. $g(x) = |x|$

12. $g(x) = \sqrt{x}$

13. $f(t)$ is the height of a football t seconds after kickoff.

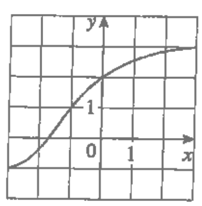
14. $f(t)$ is your height at age t.

15. Assume that f is a one-to-one function.
(a) If $f(6) = 17$, what is $f^{-1}(17)$?
(b) If $f^{-1}(3) = 2$, what is $f(2)$?

16. If $f(x) = x^5 + x^3 + x$, find $f^{-1}(3)$ and $f(f^{-1}(2))$.

17. If $g(x) = 3 + x + e^x$, find $g^{-1}(4)$.

18. The graph of f is given.
(a) Why is f one-to-one?
(b) What are the domain and range of f^{-1}?
(c) What is the value of $f^{-1}(2)$?
(d) Estimate the value of $f^{-1}(0)$.



19. The formula $C = \frac{5}{9}(F - 32)$, where $F \geqslant -459.67$, expresses the Celsius temperature C as a function of the Fahrenheit temperature F. Find a formula for the inverse function and interpret it. What is the domain of the inverse function?

20. In the theory of relativity, the mass of a particle with speed v is

$$m = f(v) = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

where m_0 is the rest mass of the particle and c is the speed of light in a vacuum. Find the inverse function of f and explain its meaning.

21–26 Find a formula for the inverse of the function.

21. $f(x) = 1 + \sqrt{2 + 3x}$

22. $f(x) = \dfrac{4x - 1}{2x + 3}$

23. $f(x) = e^{2x-1}$

24. $y = x^2 - x, \quad x \geqslant \frac{1}{2}$

25. $y = \ln(x + 3)$

26. $y = \dfrac{e^x}{1 + 2e^x}$

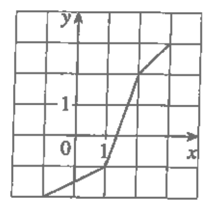
27–28 Find an explicit formula for f^{-1} and use it to graph f^{-1}, f, and the line $y = x$ on the same screen. To check your work, see whether the graphs of f and f^{-1} are reflections about the line.

27. $f(x) = x^4 + 1, \quad x \geqslant 0$

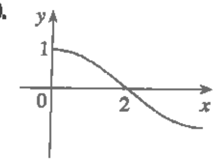
28. $f(x) = 2 - e^x$

29–30 Use the given graph of f to sketch the graph of f^{-1}.

29.



30.



31. Let $f(x) = \sqrt{1 - x^2}$, $0 \leqslant x \leqslant 1$.
(a) Find f^{-1}. How is it related to f?
(b) Identify the graph of f and explain your answer to part (a).

32. Let $g(x) = \sqrt[3]{1 - x^3}$.
(a) Find g^{-1}. How is it related to g?
(b) Graph g. How do you explain your answer to part (a)?

33. (a) How is the logarithmic function $y = \log_a x$ defined?
(b) What is the domain of this function?
(c) What is the range of this function?
(d) Sketch the general shape of the graph of the function $y = \log_a x$ if $a > 1$.

34. (a) What is the natural logarithm?
(b) What is the common logarithm?
(c) Sketch the graphs of the natural logarithm function and the natural exponential function with a common set of axes.

35–38 Find the exact value of each expression.

35. (a) $\log_5 125$ (b) $\log_3(\frac{1}{27})$

36. (a) $\ln(1/e)$ (b) $\log_{10}\sqrt{10}$

37. (a) $\log_2 6 - \log_2 15 + \log_2 20$
(b) $\log_3 100 - \log_3 18 - \log_3 50$

38. (a) $e^{-2\ln 5}$ (b) $\ln(\ln e^{e^{10}})$

39–41 Express the given quantity as a single logarithm.

39. $\ln 5 + 5\ln 3$

40. $\ln(a + b) + \ln(a - b) - 2\ln c$

41. $\frac{1}{3}\ln(x + 2)^3 + \frac{1}{2}[\ln x - \ln(x^2 + 3x + 2)^2]$

42. Use Formula 10 to evaluate each logarithm correct to six decimal places.
(a) $\log_{12} 10$ (b) $\log_2 8.4$

43–44 Use Formula 10 to graph the given functions on a common screen. How are these graphs related?

43. $y = \log_{1.5} x, \quad y = \ln x, \quad y = \log_{10} x, \quad y = \log_{50} x$

44. $y = \ln x, \quad y = \log_{10} x, \quad y = e^x, \quad y = 10^x$

45. Suppose that the graph of $y = \log_2 x$ is drawn on a coordinate grid where the unit of measurement is a centimeter. How many kilometers to the right of the origin do we have to move before the height of the curve reaches 1 m?

46. Compare the functions $f(x) = x^{0.1}$ and $g(x) = \ln x$ by graphing both f and g in several viewing rectangles. When does the graph of f finally surpass the graph of g?

47–48 Make a rough sketch of the graph of each function. Do not use a calculator. Just use the graphs given in Figures 12 and 13 and, if necessary, the transformations of Section 1.3.

47. (a) $y = \log_{10}(x + 5)$ (b) $y = -\ln x$

48. (a) $y = \ln(-x)$ (b) $y = \ln|x|$

49–50 (a) What are the domain and range of f?
(b) What is the x-intercept of the graph of f?
(c) Sketch the graph of f.

49. $f(x) = \ln x + 2$

50. $f(x) = \ln(x - 1) - 1$

51–54 Solve each equation for x.

51. (a) $2 \ln x = 1$ (b) $e^{-x} = 5$

52. (a) $e^{2x+3} - 7 = 0$ (b) $\ln(5 - 2x) = -3$

53. (a) $2^{x-5} = 3$ (b) $\ln x + \ln(x - 1) = 1$

54. (a) $\ln(\ln x) = 1$ (b) $e^{ax} = Ce^{bx}$, where $a \neq b$

55–56 Solve each inequality for x.

55. (a) $\ln x < 0$ (b) $e^x > 5$

56. (a) $1 < e^{3x-1} < 2$ (b) $1 - 2 \ln x < 3$

57. (a) Find the domain of $f(x) = \ln(e^x - 3)$.
(b) Find f^{-1} and its domain.

58. (a) What are the values of $e^{\ln 300}$ and $\ln(e^{300})$?
(b) Use your calculator to evaluate $e^{\ln 300}$ and $\ln(e^{300})$. What do you notice? Can you explain why the calculator has trouble?

CAS **59.** Graph the function $f(x) = \sqrt{x^3 + x^2 + x + 1}$ and explain why it is one-to-one. Then use a computer algebra system to find an explicit expression for $f^{-1}(x)$. (Your CAS will produce three possible expressions. Explain why two of them are irrelevant in this context.)

CAS **60.** (a) If $g(x) = x^6 + x^4, x \geq 0$, use a computer algebra system to find an expression for $g^{-1}(x)$.
(b) Use the expression in part (a) to graph $y = g(x)$, $y = x$, and $y = g^{-1}(x)$ on the same screen.

61. If a bacteria population starts with 100 bacteria and doubles every three hours, then the number of bacteria after t hours is $n = f(t) = 100 \cdot 2^{t/3}$. (See Exercise 29 in Section 1.5.)
(a) Find the inverse of this function and explain its meaning.
(b) When will the population reach 50,000?

62. When a camera flash goes off, the batteries immediately begin to recharge the flash's capacitor, which stores electric charge given by

$$Q(t) = Q_0(1 - e^{-t/a})$$

(The maximum charge capacity is Q_0 and t is measured in seconds.)
(a) Find the inverse of this function and explain its meaning.
(b) How long does it take to recharge the capacitor to 90% of capacity if $a = 2$?

63–68 Find the exact value of each expression.

63. (a) $\sin^{-1}(\sqrt{3}/2)$ (b) $\cos^{-1}(-1)$

64. (a) $\tan^{-1}(1/\sqrt{3})$ (b) $\sec^{-1} 2$

65. (a) $\arctan 1$ (b) $\sin^{-1}(1/\sqrt{2})$

66. (a) $\cot^{-1}(-\sqrt{3})$ (b) $\arccos(-\frac{1}{2})$

67. (a) $\tan(\arctan 10)$ (b) $\sin^{-1}(\sin(7\pi/3))$

68. (a) $\tan(\sec^{-1} 4)$ (b) $\sin(2 \sin^{-1}(\frac{3}{5}))$

69. Prove that $\cos(\sin^{-1} x) = \sqrt{1 - x^2}$.

70–72 Simplify the expression.

70. $\tan(\sin^{-1} x)$

71. $\sin(\tan^{-1} x)$

72. $\cos(2 \tan^{-1} x)$

73–74 Graph the given functions on the same screen. How are these graphs related?

73. $y = \sin x, \ -\pi/2 \leq x \leq \pi/2; \quad y = \sin^{-1} x; \quad y = x$

74. $y = \tan x, \ -\pi/2 < x < \pi/2; \quad y = \tan^{-1} x; \quad y = x$

75. Find the domain and range of the function

$$g(x) = \sin^{-1}(3x + 1)$$

76. (a) Graph the function $f(x) = \sin(\sin^{-1} x)$ and explain the appearance of the graph.
(b) Graph the function $g(x) = \sin^{-1}(\sin x)$. How do you explain the appearance of this graph?

77. (a) If we shift a curve to the left, what happens to its reflection about the line $y = x$? In view of this geometric principle, find an expression for the inverse of $g(x) = f(x + c)$, where f is a one-to-one function.
(b) Find an expression for the inverse of $h(x) = f(cx)$, where $c \neq 0$.

1.6 Inverse Functions and Logarithms

1. (a) See Definition 1. (b) It must pass the Horizontal Line Test.

3. f is not one-to-one because $2 \neq 6$, but $f(2) = 2.0 = f(6)$.

5. We could draw a horizontal line that intersects the graph in more than one point. Thus, by the Horizontal Line Test, the function is not one-to-one.

7. No horizontal line intersects the graph more than once. Thus, by the Horizontal Line Test, the function is one-to-one.

9. The graph of $f(x) = \frac{1}{2}(x+5)$ is a line with slope $\frac{1}{2}$. It passes the Horizontal Line Test, so f is one-to-one.
Algebraic solution: If $x_1 \neq x_2$, then $x_1 + 5 \neq x_2 + 5 \;\Rightarrow\; \frac{1}{2}(x_1+5) \neq \frac{1}{2}(x_2+5) \;\Rightarrow\; f(x_1) \neq f(x_2)$, so f is one-to-one.

11. $g(x) = |x| \;\Rightarrow\; g(-1) = 1 = g(1)$, so g is not one-to-one.

13. A football will attain every height h up to its maximum height twice: once on the way up, and again on the way down. Thus, even if t_1 does not equal t_2, $f(t_1)$ may equal $f(t_2)$, so f is not 1-1.

15. (a) Since f is 1-1, $f(6) = 17 \;\Leftrightarrow\; f^{-1}(17) = 6$. (b) Since f is 1-1, $f^{-1}(3) = 2 \;\Leftrightarrow\; f(2) = 3$.

17. First, we must determine x such that $g(x) = 4$. By inspection, we see that if $x = 0$, then $g(x) = 4$. Since g is 1-1 (g is an increasing function), it has an inverse, and $g^{-1}(4) = 0$.

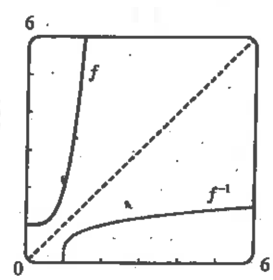
19. We solve $C = \frac{5}{9}(F - 32)$ for F: $\frac{9}{5}C = F - 32 \;\Rightarrow\; F = \frac{9}{5}C + 32$. This gives us a formula for the inverse function, that is, the Fahrenheit temperature F as a function of the Celsius temperature C. $F \geq -459.67 \;\Rightarrow\; \frac{9}{5}C + 32 \geq -459.67 \;\Rightarrow\; \frac{9}{5}C \geq -491.67 \;\Rightarrow\; C \geq -273.15$, the domain of the inverse function.

21. $y = f(x) = 1 + \sqrt{2+3x} \quad (y \geq 1) \;\Rightarrow\; y - 1 = \sqrt{2+3x} \;\Rightarrow\; (y-1)^2 = 2 + 3x \;\Rightarrow\; (y-1)^2 - 2 = 3x \;\Rightarrow\; x = \frac{1}{3}(y-1)^2 - \frac{2}{3}$. Interchange x and y: $y = \frac{1}{3}(x-1)^2 - \frac{2}{3}$. So $f^{-1}(x) = \frac{1}{3}(x-1)^2 - \frac{2}{3}$. Note that the domain of f^{-1} is $x \geq 1$.

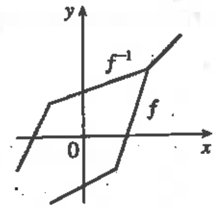
23. $y = f(x) = e^{2x-1} \;\Rightarrow\; \ln y = 2x - 1 \;\Rightarrow\; 1 + \ln y = 2x \;\Rightarrow\; x = \frac{1}{2}(1 + \ln y)$.
Interchange x and y: $y = \frac{1}{2}(1 + \ln x)$. So $f^{-1}(x) = \frac{1}{2}(1 + \ln x)$.

25. $y = f(x) = \ln(x+3) \;\Rightarrow\; x + 3 = e^y \;\Rightarrow\; x = e^y - 3$. Interchange x and y: $y = e^x - 3$. So $f^{-1}(x) = e^x - 3$.

27. $y = f(x) = x^4 + 1 \;\Rightarrow\; y - 1 = x^4 \;\Rightarrow\; x = \sqrt[4]{y-1}$ [not $\pm$ since $x \geq 0$]. Interchange x and y: $y = \sqrt[4]{x-1}$. So $f^{-1}(x) = \sqrt[4]{x-1}$. The graph of $y = \sqrt[4]{x-1}$ is just the graph of $y = \sqrt[4]{x}$ shifted right one unit. From the graph, we see that f and f^{-1} are reflections about the line $y = x$.

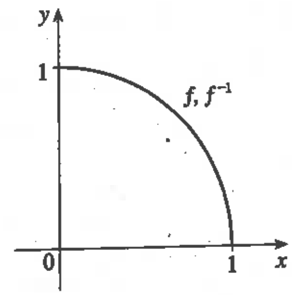


29. Reflect the graph of f about the line $y = x$. The points $(-1, -2)$, $(1, -1)$, $(2, 2)$, and $(3, 3)$ on f are reflected to $(-2, -1)$, $(-1, 1)$, $(2, 2)$, and $(3, 3)$ on f^{-1}.



31. (a) $y = f(x) = \sqrt{1 - x^2}$ $(0 \le x \le 1$ and note that $y \ge 0)$ $\Rightarrow$ $y^2 = 1 - x^2$ $\Rightarrow$ $x^2 = 1 - y^2$ $\Rightarrow$ $x = \sqrt{1 - y^2}$. So $f^{-1}(x) = \sqrt{1 - x^2}$, $0 \le x \le 1$. We see that f^{-1} and f are the same function.

(b) The graph of f is the portion of the circle $x^2 + y^2 = 1$ with $0 \le x \le 1$ and $0 \le y \le 1$ (quarter-circle in the first quadrant). The graph of f is symmetric with respect to the line $y = x$, so its reflection about $y = x$ is itself, that is, $f^{-1} = f$.



33. (a) It is defined as the inverse of the exponential function with base a, that is, $\log_a x = y \iff a^y = x$.

(b) $(0, \infty)$ (c) $\mathbb{R}$ (d) See Figure 11.

35. (a) $\log_5 125 = 3$ since $5^3 = 125$. (b) $\log_3 \frac{1}{27} = -3$ since $3^{-3} = \frac{1}{3^3} = \frac{1}{27}$.

37. (a)
$$\begin{aligned}\log_2 6 - \log_2 15 + \log_2 20 &= \log_2(\tfrac{6}{15}) + \log_2 20 && \text{[by Law 2]}\\ &= \log_2(\tfrac{6}{15} \cdot 20) && \text{[by Law 1]}\\ &= \log_2 8, \text{ and } \log_2 8 = 3 \text{ since } 2^3 = 8.\end{aligned}$$

(b)
$$\begin{aligned}\log_3 100 - \log_3 18 - \log_3 50 &= \log_3(\tfrac{100}{18}) - \log_3 50 = \log_3(\tfrac{100}{18 \cdot 50})\\ &= \log_3(\tfrac{1}{9}), \text{ and } \log_3(\tfrac{1}{9}) = -2 \text{ since } 3^{-2} = \tfrac{1}{9}.\end{aligned}$$

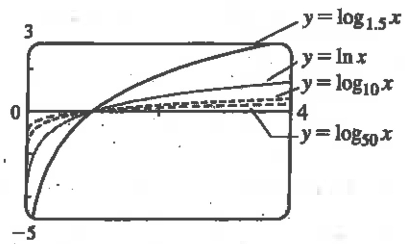
39.
$$\begin{aligned}\ln 5 + 5\ln 3 &= \ln 5 + \ln 3^5 && \text{[by Law 3]}\\ &= \ln(5 \cdot 3^5) && \text{[by Law 1]}\\ &= \ln 1215\end{aligned}$$

41.
$$\begin{aligned}\tfrac{1}{3}\ln(x+2)^3 + \tfrac{1}{2}\left[\ln x - \ln(x^2+3x+2)^2\right] &= \ln[(x+2)^3]^{1/3} + \tfrac{1}{2}\ln\frac{x}{(x^2+3x+2)^2} && \text{[by Laws 3, 2]}\\ &= \ln(x+2) + \ln\frac{\sqrt{x}}{x^2+3x+2} && \text{[by Law 3]}\\ &= \ln\frac{(x+2)\sqrt{x}}{(x+1)(x+2)} && \text{[by Law 1]}\\ &= \ln\frac{\sqrt{x}}{x+1}\end{aligned}$$

Note that since $\ln x$ is defined for $x > 0$, we have $x + 1$, $x + 2$, and $x^2 + 3x + 2$ all positive, and hence their logarithms are defined.

43. To graph these functions, we use $\log_{1.5} x = \dfrac{\ln x}{\ln 1.5}$ and $\log_{50} x = \dfrac{\ln x}{\ln 50}$.

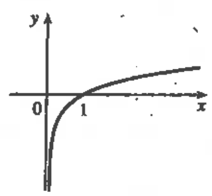
These graphs all approach $-\infty$ as $x \to 0^+$, and they all pass through the point $(1, 0)$. Also, they are all increasing, and all approach ∞ as $x \to \infty$. The functions with larger bases increase extremely slowly, and the ones with smaller bases do so somewhat more quickly. The functions with large bases approach the y-axis more closely as $x \to 0^+$.



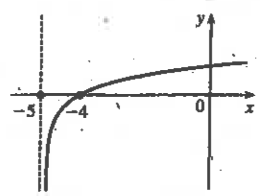
45. 1 m = 100 cm, so we need x such that $\log_2 x = 100 \;\Leftrightarrow\; x = 2^{100}$ cm. In kilometers, this is

$2^{100}\text{ cm} \cdot \dfrac{1\text{ km}}{10^5\text{ cm}} \approx 1.27 \times 10^{25}$ km.

47. (a) Shift the graph of $y = \log_{10} x$ five units to the left to obtain the graph of $y = \log_{10}(x + 5)$. Note the vertical asymptote of $x = -5$.

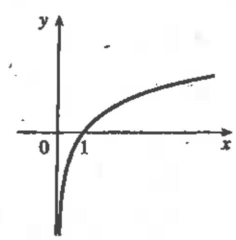


$y = \log_{10} x$

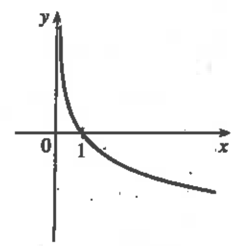


$y = \log_{10}(x + 5)$

(b) Reflect the graph of $y = \ln x$ about the x-axis to obtain the graph of $y = -\ln x$.



$y = \ln x$

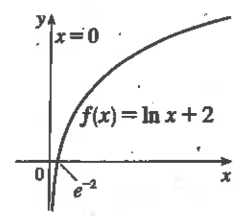


$y = -\ln x$

49. (a) The domain of $f(x) = \ln x + 2$ is $x > 0$ and the range is $\mathbb{R}$.

(b) $y = 0 \;\Rightarrow\; \ln x + 2 = 0 \;\Rightarrow\; \ln x = -2 \;\Rightarrow\; x = e^{-2}$

(c) We shift the graph of $y = \ln x$ two units upward.



51. (a) $2 \ln x = 1 \;\Rightarrow\; \ln x = \frac{1}{2} \;\Rightarrow\; x = e^{1/2} = \sqrt{e}$

(b) $e^{-x} = 5 \;\Rightarrow\; -x = \ln 5 \;\Rightarrow\; x = -\ln 5$

53. (a) $2^{x-5} = 3 \;\Leftrightarrow\; \log_2 3 = x - 5 \;\Leftrightarrow\; x = 5 + \log_2 3$.

Or: $2^{x-5} = 3 \;\Leftrightarrow\; \ln(2^{x-5}) = \ln 3 \;\Leftrightarrow\; (x - 5)\ln 2 = \ln 3 \;\Leftrightarrow\; x - 5 = \dfrac{\ln 3}{\ln 2} \;\Leftrightarrow\; x = 5 + \dfrac{\ln 3}{\ln 2}$

(b) $\ln x + \ln(x - 1) = \ln(x(x - 1)) = 1 \;\Leftrightarrow\; x(x - 1) = e^1 \;\Leftrightarrow\; x^2 - x - e = 0$. The quadratic formula (with $a = 1$, $b = -1$, and $c = -e$) gives $x = \frac{1}{2}\left(1 \pm \sqrt{1 + 4e}\right)$, but we reject the negative root since the natural logarithm is not defined for $x < 0$. So $x = \frac{1}{2}\left(1 + \sqrt{1 + 4e}\right)$.

55. (a) $\ln x < 0 \;\Rightarrow\; x < e^0 \;\Rightarrow\; x < 1$. Since the domain of $f(x) = \ln x$ is $x > 0$, the solution of the original inequality is $0 < x < 1$.

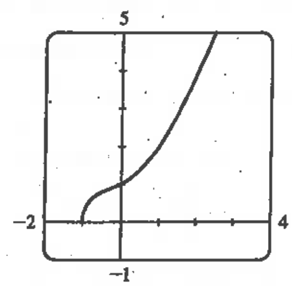
(b) $e^x > 5 \;\Rightarrow\; \ln e^x > \ln 5 \;\Rightarrow\; x > \ln 5$

57. (a) We must have $e^x - 3 > 0 \;\Rightarrow\; e^x > 3 \;\Rightarrow\; x > \ln 3$. Thus, the domain of $f(x) = \ln(e^x - 3)$ is $(\ln 3, \infty)$.

(b) $y = \ln(e^x - 3) \;\Rightarrow\; e^y = e^x - 3 \;\Rightarrow\; e^x = e^y + 3 \;\Rightarrow\; x = \ln(e^y + 3)$, so $f^{-1}(x) = \ln(e^x + 3)$.

Now $e^x + 3 > 0 \;\Rightarrow\; e^x > -3$, which is true for any real x, so the domain of f^{-1} is $\mathbb{R}$.

59. We see that the graph of $y = f(x) = \sqrt{x^3 + x^2 + x + 1}$ is increasing, so f is 1-1.



Enter $x = \sqrt{y^3 + y^2 + y + 1}$ and use your CAS to solve the equation for y.

Using Derive, we get two (irrelevant) solutions involving imaginary expressions, as well as one which can be simplified to the following:

$$y = f^{-1}(x) = -\frac{\sqrt[3]{4}}{6}\left(\sqrt[3]{D - 27x^2 + 20} - \sqrt[3]{D + 27x^2 - 20} + \sqrt[3]{2}\right)$$

where $D = 3\sqrt{3}\,\sqrt{27x^4 - 40x^2 + 16}$.

Maple and Mathematica each give two complex expressions and one real expression, and the real expression is equivalent to that given by Derive. For example, Maple's expression simplifies to $\dfrac{1}{6}\,\dfrac{M^{2/3} - 8 - 2M^{1/3}}{2M^{1/3}}$, where $M = 108x^2 + 12\sqrt{48 - 120x^2 + 81x^4} - 80$.

61. (a) $n = f(t) = 100 \cdot 2^{t/3} \;\Rightarrow\; \dfrac{n}{100} = 2^{t/3} \;\Rightarrow\; \log_2\left(\dfrac{n}{100}\right) = \dfrac{t}{3} \;\Rightarrow\; t = 3\log_2\left(\dfrac{n}{100}\right)$. Using formula (10), we can write this as $t = f^{-1}(n) = 3 \cdot \dfrac{\ln(n/100)}{\ln 2}$. This function tells us how long it will take to obtain n bacteria (given the number n).

(b) $n = 50{,}000 \;\Rightarrow\; t = f^{-1}(50{,}000) = 3 \cdot \dfrac{\ln\left(\frac{50{,}000}{100}\right)}{\ln 2} = 3\left(\dfrac{\ln 500}{\ln 2}\right) \approx 26.9$ hours

63. (a) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$ since $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ and $\frac{\pi}{3}$ is in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

(b) $\cos^{-1}(-1) = \pi$ since $\cos\pi = -1$ and π is in $[0, \pi]$.

65. (a) $\arctan 1 = \frac{\pi}{4}$ since $\tan\frac{\pi}{4} = 1$ and $\frac{\pi}{4}$ is in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

(b) $\sin^{-1}\frac{1}{\sqrt{2}} = \frac{\pi}{4}$ since $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ and $\frac{\pi}{4}$ is in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

67. (a) In general, $\tan(\arctan x) = x$ for any real number x. Thus, $\tan(\arctan 10) = 10$.

(b) $\sin^{-1}\left(\sin\frac{7\pi}{3}\right) = \sin^{-1}\left(\sin\frac{\pi}{3}\right) = \sin^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{3}$ since $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ and $\frac{\pi}{3}$ is in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

[Recall that $\frac{7\pi}{3} = \frac{\pi}{3} + 2\pi$ and the sine function is periodic with period 2π.]

69. Let $y = \sin^{-1} x$. Then $-\frac{\pi}{2} \le y \le \frac{\pi}{2} \;\Rightarrow\; \cos y \ge 0$, so $\cos(\sin^{-1} x) = \cos y = \sqrt{1 - \sin^2 y} = \sqrt{1 - x^2}$.

71. Let $y = \tan^{-1} x$. Then $\tan y = x$, so from the triangle we see that

$$\sin(\tan^{-1} x) = \sin y = \frac{x}{\sqrt{1 + x^2}}.$$