

1.5 Exercises

1–4 Use the Law of Exponents to rewrite and simplify the expression.

- (a) $\frac{4^{-3}}{2^{-8}}$ (b) $\frac{1}{\sqrt[3]{x^4}}$
- (a) $8^{4/3}$ (b) $x(3x^2)^3$
- (a) $b^8(2b)^4$ (b) $\frac{(6y^3)^4}{2y^5}$
- (a) $\frac{x^{2n} \cdot x^{3n-1}}{x^{n+2}}$ (b) $\frac{\sqrt{a}\sqrt{b}}{\sqrt[3]{ab}}$

- (a) Write an equation that defines the exponential function with base $a > 0$.
 (b) What is the domain of this function?
 (c) If $a \neq 1$, what is the range of this function?
 (d) Sketch the general shape of the graph of the exponential function for each of the following cases.
 (i) $a > 1$ (ii) $a = 1$ (iii) $0 < a < 1$
- (a) How is the number e defined?
 (b) What is an approximate value for e ?
 (c) What is the natural exponential function?

 7–10 Graph the given functions on a common screen. How are these graphs related?

- $y = 2^x$, $y = e^x$, $y = 5^x$, $y = 20^x$
- $y = e^x$, $y = e^{-x}$, $y = 8^x$, $y = 8^{-x}$
- $y = 3^x$, $y = 10^x$, $y = (\frac{1}{3})^x$, $y = (\frac{1}{10})^x$
- $y = 0.9^x$, $y = 0.6^x$, $y = 0.3^x$, $y = 0.1^x$

11–16 Make a rough sketch of the graph of the function. Do not use a calculator. Just use the graphs given in Figures 3 and 13 and, if necessary, the transformations of Section 1.3.

- $y = 10^{x+2}$ (12) $y = (0.5)^x - 2$
- $y = -2^{-x}$ (14) $y = e^{|x|}$
- $y = 1 - \frac{1}{2}e^{-x}$ (16) $y = 2(1 - e^x)$

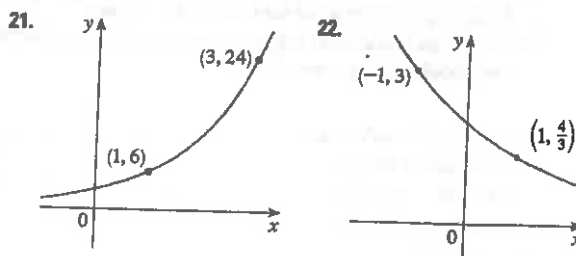
- Starting with the graph of $y = e^x$, write the equation of the graph that results from
 (a) shifting 2 units downward
 (b) shifting 2 units to the right
 (c) reflecting about the x -axis
 (d) reflecting about the y -axis
 (e) reflecting about the x -axis and then about the y -axis

- Starting with the graph of $y = e^x$, find the equation of the graph that results from
 (a) reflecting about the line $y = 4$
 (b) reflecting about the line $x = 2$

19–20 Find the domain of each function.



- (a) $f(x) = \frac{1 - e^{x^2}}{1 - e^{1-x^2}}$ (b) $f(x) = \frac{1+x}{e^{\cos x}}$
- (a) $g(t) = \sin(e^{-t})$ (b) $g(t) = \sqrt{1-2^t}$

21–22 Find the exponential function $f(x) = Ca^x$ whose graph is given.



- If $f(x) = 5^x$, show that

$$\frac{f(x+h) - f(x)}{h} = 5^x \left(\frac{5^h - 1}{h} \right)$$

- Suppose you are offered a job that lasts one month. Which of the following methods of payment do you prefer?
 I. One million dollars at the end of the month.
 II. One cent on the first day of the month, two cents on the second day, four cents on the third day, and, in general, 2^{n-1} cents on the n th day.
- Suppose the graphs of $f(x) = x^2$ and $g(x) = 2^x$ are drawn on a coordinate grid where the unit of measurement is 1 cm. Show that, at a distance 1 m to the right of the origin, the height of the graph of f is 100 m but the height of the graph of g is more than 10^{25} km.
-  Compare the functions $f(x) = x^5$ and $g(x) = 5^x$ by graphing both functions in several viewing rectangles. Find all points of intersection of the graphs correct to one decimal place. Which function grows more rapidly when x is large?
-  Compare the functions $f(x) = x^{10}$ and $g(x) = e^x$ by graphing both f and g in several viewing rectangles. When does the graph of g finally surpass the graph of f ?

 Graphing calculator or computer required

1. Homework Hints available at stewartcalculus.com

28. Use a graph to estimate the values of x such that $e^x > 1,000,000,000$.
29. Under ideal conditions a certain bacteria population is known to double every three hours. Suppose that there are initially 100 bacteria.
- What is the size of the population after 15 hours?
 - What is the size of the population after t hours?
 - Estimate the size of the population after 20 hours.
30. A bacterial culture starts with 500 bacteria and doubles in size every half hour.
- How many bacteria are there after 3 hours?
 - How many bacteria are there after t hours?
 - How many bacteria are there after 40 minutes?
31. Use a graphing calculator with exponential regression capability to model the population of the world with the data from 1950 to 2010 in Table 1 on page 54. Use the model to estimate the population in 1993 and to predict the population in the year 2020.
32. The table gives the population of Malaysia, in millions, for the years 1950–2000. Use a graphing calculator with exponential regression capability to model the Malaysian population since 1950. Use the model to estimate the population in 1975 and to predict the population in the years 2010 and 2020.

Year	Population	Year	Population
1950	6.1	1980	13.8
1955	7.0	1985	15.7
1960	8.1	1990	17.8
1965	9.5	1995	20.4
1970	10.9	2000	23.0
1975	12.3		

33. If you graph the function

$$f(x) = \frac{1 - e^{1/x}}{1 + e^{1/x}}$$

you'll see that f appears to be an odd function. Prove it.

34. Graph several members of the family of functions

$$f(x) = \frac{1}{1 + ae^{bx}}$$

where $a > 0$. How does the graph change when b changes? How does it change when a changes?

1.6 Inverse Functions and Logarithms

Table 1 gives data from an experiment in which a bacteria culture started with 100 bacteria in a limited nutrient medium; the size of the bacteria population was recorded at hourly intervals. The number of bacteria N is a function of the time t : $N = f(t)$.

Suppose, however, that the biologist changes her point of view and becomes interested in the time required for the population to reach various levels. In other words, she is thinking of t as a function of N . This function is called the *inverse function* of f , denoted by f^{-1} , and read " f inverse." Thus $t = f^{-1}(N)$ is the time required for the population level to reach N . The values of f^{-1} can be found by reading Table 1 from right to left or by consulting Table 2. For instance, $f^{-1}(550) = 6$ because $f(6) = 550$.

TABLE 1 N as a function of t

t (hours)	$N = f(t)$ = population at time t
0	100
1	168
2	259
3	358
4	445
5	509
6	550
7	573
8	586

TABLE 2 t as a function of N

N	$t = f^{-1}(N)$ = time to reach N bacteria
100	0
168	1
259	2
358	3
445	4
509	5
550	6
573	7
586	8